Categorizing carrier-byproduct metal pairs to assess materials criticality - Focus on price elasticity of photovoltaics related metals

by

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B.S. Materials Science
Università degli Studi di Padova, 2014

Submitted to the UPC Department of Industrial Engineering for the Partial Fulfillment of the Requirements for the Degree

of

Master of Science in Environomical Pathways for Sustainable Energy Systems

at the Universitat Politècnica de Catalunya and
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July 2016

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Abstract

Clean energy technologies, such as photovoltaics, wind turbines, efficient lighting systems and electric vehicles, are key players in moving towards a sustainable future. However, each of these technologies relies on significant use of specialized metals, raising materials criticality concerns as the demand for these devices (and the materials they contain) increases. In addition to low ore concentrations and missing trade at major public exchanges, one of the main sources of concern is their byproduct nature. In many cases, in fact, such metals are obtained as minor products of more abundant materials, which are referred as carrier metals. Some examples include cadmium, gallium, germanium, indium, selenium and tellurium.

The point discussed within the current literature is that the maximum supply potential of byproduct metals is limited by the supply of carrier metals. Moreover, price inelasticity of byproducts (supply not responding to price changes) have been cited as proof of such constraint.

The first part of the present work aims to categorize forty-seven carrier-byproduct pairs according to byproduct fraction and value ratio to assess criticality. Both a qualitative and a quantitative evaluation of the obtained matrix are performed, using hierarchical clustering for the quantitative analysis.

The objective of the second part of the study is to verify whether the supply of byproduct metals is inelastic and, if so, understand whether this is caused by carriers' supply limiting byproduct supply. Indium and tellurium are used as case studies. The selection of those elements is driven by their wide application in clean energy technologies, from thin-film photovoltaics cells to nuclear power control rods. Econometric analyses including ordinary least squares, autoregressive distributed lag and two stages least square models are performed.

Five similar groups are identified in both the qualitative and quantitative evaluation. Each cluster consists of pairs with similar overall criticality that impact market players in a similar way. Two groups are found to be critical for consumers, one for producers, one for both and one for none.

For what concerns supply inelasticity, econometric analyses suggest that both indium and tellurium are price inelastic to supply. However, while in the first case the reason is found to be limitation of the carrier metal, for tellurium this seems not to be the case. Non-transparent trading, monopolistic character of supply and other factors are indeed expected to be the major causes.
Future work will investigate additional byproduct pairs using similar econometric models. The final aim of the project is to produce outcomes which will be useful for decision making of both metals producers and consumers.

Acknowledgments

I would like to thank my supervisors, Professor Elsa Olivetti from MIT and Professor César Valderrama from UPC for their encouragement and inspiring guidance throughout the development of my thesis. I would also like to thank my friend Irene, for being always ready to listen to me and provide some useful feedbacks on my work. But most of all, I would like to thank my research fellow, Xinkai. Besides helping me with the more technical parts and the editing, he became a real friend. I really enjoyed my time here in Boston and part of if it's thank to him.

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Nomenclature

2SLS: Two stages Least Squares

ARDL: Autoregressive Distributed Lag

BGS: British Geological Survey

BIC: Bayesian Information Criterion

c-Si: Crystalline Silicon

GDP: Gross Domestic Product

HHI: Herfindahl–Hirschman Index

IEA: International Energy Agency

IP: Industrial Prouction

IV: Instrumental Variable

LME: London Metal Exchange

NRC: National Research Council

OECD: Organization for Economic Co-operation and Development

OLS: Ordinary Least Squares

PGM: Platinum Group Metals

PV: Photovoltaic

RRE: Rare Earth Element

S&P 500: Standard and Poor's 500

USGS: United States Geological Survey

VAR: Vector Autoregression

1 Introduction

Due to the rapid growth of energy demand in emerging economies, the willingness of governments of minimizing imports of fossil fuels by diversifying their energy sources and the increased awareness about climate issues, a clean energy revolution is taking place. All around the world, photovoltaic (PV) and wind power systems are integrated to the grid at a rate of tens GW of added capacity per year (Fraunhofer Institute for Solar Energy Systems 2016; GWEC-Global Wind Energy Council 2016). Large biomass fired combined cycles are implemented and 63 third-generation nuclear power plants are under construction (IAEA-International Atomic Energy Agency n.d.).

One of the main differences between these power generation technologies and those built in the past couple of centuries, are the materials involved (Resnick Institute 2011). While large fossil fuel power plants mostly imply the use of few, widely abundant, metals, i.e. steel, copper and others, sustainable systems make use of a much broader array of metals. Photovoltaic is a classic example. The four most widely used type of solar cells, namely crystalline silicon, amorphous silicon, cadmium-telluride (CdTe) and copper-indium-gallium-selenide (CIGS) cells, use at least 11 metals, four of which have a crustal abundance lower than 0.1% (Moss et al. 2011; Wikipedia n.d.). This includes indium, selenium and tellurium. Wind energy is also dependent on specific elements, besides most common steel, copper, nickel, chromium, manganese and molybdenum. In particular, neodymium and dysprosium, two rare earth elements (REE), are involved in the manufacturing of magnets used in permanent magnet generators (Moss et al. 2011). Moreover, third generation nuclear power plants rely on a broad class of materials. Alongside with steel, copper, cobalt, lead, nickel, chromium, tin and molybdenum, minor metals are required in the construction of the reactors and all other components. In particular, significant quantities of cadmium, hafnium, silver, titanium, tungsten, vanadium, yttrium and zirconium are used (Moss et al. 2011). But the energy (r)evolution, as described by Greenpeace and other organizations in the "Sustainable Energy Outlook" reports, is not limited to electricity generating facilities. New storage devices need to be implemented, mobilization electrified and energy efficiency measures adopted (Greenpeace

International, Global Wind Energy Council, and Solar Power Europe 2015). All these technologies rely on a massive use of specialized metals. Lithium, for example, is massively employed in energy storage devices, from mobile phones batteries to megawatts-size battery banks. Li and Li-ion batteries, are increasingly gaining market share over other types of electricity storage devices and a huge amount is expected to be consumed in the coming years (International Renewable Energy Agency 2015). Yttrium, cerium, europium and other REEs, on the other hand, are essential for high efficiency lighting systems, being used in both fluorescent bulbs and LEDs technologies (US Department of Energy 2011). Platinum group metals (PGM) also are crucial in a great variety of energy applications, with catalyst in fuel cells as one of the most important (Resnick Institute 2011).

Concerns have been raised about future availability of some of these specialized metals. In order to sustainably meet the energetic needs of a constantly increasing number of people, in fact, orders of magnitudes of clean energy systems need to be installed (Resnick Institute 2011). Dozens of studies have been published focusing on metals requirements of new clean technologies and different scenarios built. Graedel et al. from Yale studied selenium and tellurium anthropogenic cycles (Kavlak and Graedel 2013a, 2013b). Kirchain et al. from MIT Materials System Laboratory examined Platinum and REE availability for clean technologies and the automotive sector respectively (Alonso, Field, and Kirchain 2012; Alonso et al. 2012). Frenzel et al. from the Helmholtz-Zentrum Dresden-Rossendorf worked on gallium and germanium geological availability (Frenzel, Ketris, and Gutzmer 2013; Frenzel and Seifert 2016). Speirs et al. studied lithium availability for electric vehicles (Speirs et al. 2014). And the list goes on.

Each study identifies one or more issues which are believed critical for the studied metal. In some cases, geological abundance is identified as the bottleneck (Vikström, Davidsson, and Höök 2013), in some geo-political issues seemed to matter the most (Alonso et al. 2012), in others the fact that a metal is mined as a byproduct of more common materials causes concerns (Zweibel 2010). A combination of all these possible causes of concern is what is known as metal criticality. A short overview of the history behind this term is provided in chapter two, where Graedel's comprehensive methodology for metals criticality analysis is illustrated (Graedel et al. 2012).

One of the key aspects of metals criticality related with clean energy technologies is the carrier-byproduct relation. In this contest, byproducts are metals which are mined as a secondary product of one or multiple more abundant materials. The main products are referred as carriers or hosts. The carrier-byproduct relationship has been cited as being a cause of high supply criticality, given that the maximum supply potential of the byproduct metal may be limited by its carriers (Graedel et al. 2015). More specifically, supply of byproduct metals not able to respond quickly to changes in the price (price inelasticity of supply), it’s a widely accepted assumption to justify byproduct constraints caused by the carriers.

The objective of this work is to assess the impact of byproduct mining as indicator of materials criticality, in particular focusing on photovoltaics related metals. First, carrier-byproduct pairs categorized, according to byproduct fraction and value ratio. Secondly, supply price inelasticity of byproducts is evaluated for some metals in order to verify the widely used assumption cited above. In particular, PV related metals are used as case studies. The questions to be answered are two:

“Can different carrier-byproduct pair be categorized, according to market behaviors?”

“Is supply limitation set by carrier metals the only reason causing byproduct metals supply to be inelastic to changes in price?”

Detailed explanation of the carrier-byproduct dynamics as well as of PV-related metals are provided in chapter two, while the methodology developed is presented in the chapter three. Categorization of different carrier-byproduct pairs is reported in chapter four, while the case studies on indium and tellurium are presented in chapter five. Conclusions and future works are summarized in chapter six.

Last but not least, a term-clarification. The work is mainly focused on metals, but some metalloids are also considered, i.e. selenium, tellurium, etc. For convenience, the term “metal” is used to refer both at metals and metalloids.

2 Background

2.1 Metal Criticality

Prior to the first half of the 20th century, depletion of materials resources was not considered a possible issue and the topic was not discussed. Things changed in 1952 when the “Paley Report” was published (Paley et al. 1952). For the first time, scientists stated that resource limitation was, in fact, possible. Then was the cobalt crisis of the 70’s, which showed that Paley report’s concerns may be correct. Due to political instability in Zaire, which at the time accounted for around 40% of world production, a cobalt shortage occurred, followed by a 380% increase of price. This led to a revision of cobalt end use products, sourcing routes and even national policies (Alonso, Gregory, and Field 2007).

More recently, attentions have been drawn on Rare Earth Elements (REE). A decrease of exports from China, which accounts for more than 80% of world production (US Geological Survey 2016), resulted in technological disruptions (Alonso et al. 2012). Many institutions around the globe started to conduct materials supply risk analysis and the field of materials criticality was shaped.

2.1.1 Literature Review on Materials Criticality

The first comprehensive material criticality study is by many considered the “Minerals, Critical Minerals, and the U.S. Economy” report, published by the National Research Council (NRC) in 2007 (National Research Council 2007). Materials criticality is evaluated in a 2D matrix, which includes importance of uses (named “impact to supply restriction”) and availability (named “supply restriction”) as the two axes. Figure 1 provides a schematic representation of the proposed matrix.

NRC identifies two main groups of critical materials: platinum group metals and rare earth elements. In addition, indium, manganese and niobium are also positioned in the critical portion of the matrix (top-right corner).

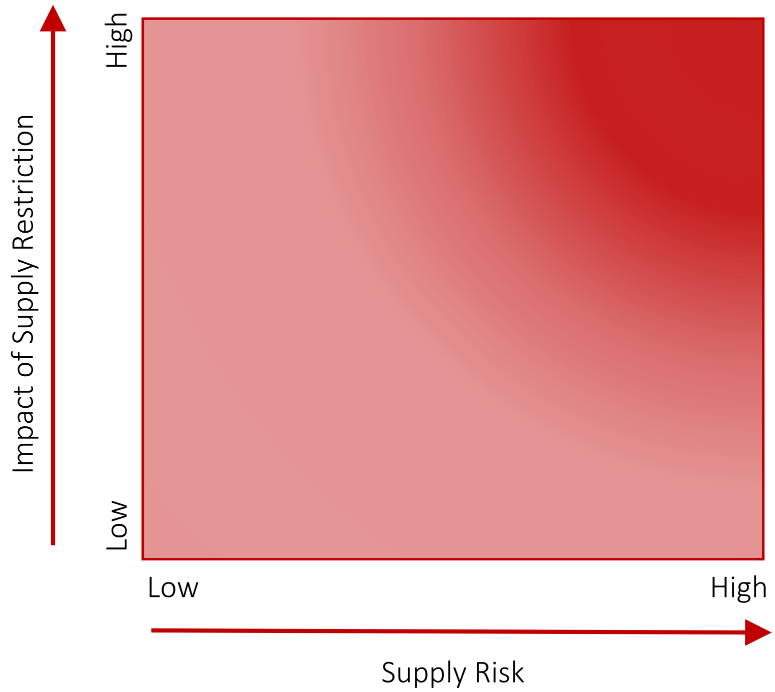


Figure 1: Criticality matrix proposed by NRC in (National Research Council 2007)

Soon after the publication of the NRC report, materials criticality gains more and more attention and other institutes develop similar matrices. In few years, a broad range of reports are published (Alonso et al. 2012; British Geological Survey 2012, 2015; Buchert, Shuler, and Bleher 2009; European Commission 2014; Moss et al. 2011; Resnick Institute 2011; Resource Efficiency Knowledge Transfer Network n.d.; US Department of Energy 2011). Each study focuses on specific elements or compounds depending on the scope, geographical considerations (national, continental or global) and organizational level (private or public company). As a consequence, most of the works are not directly comparable between each other. A metal may be considered critical under specific conditions, while raise no concerns if different constraints are considered. Nevertheless, there is an increasing number of metals which always or almost end up in the most critical part of the matrices.

The results of some of the main and broader studies are summarized in the Table 1. Red indicates high criticality, yellow medium and green low.

Table 1: Summary of results of seven metal criticality related studies

Organization	US National Research Council (National Research Council 2007)	Oakdene Hollins (Resource Efficiency Knowledge Transfer Network n.d.)	Oko Institute (Buchert, Shuler, and Bleher 2009)	EU Commission[1] (Moss et al. 2011)	US Department of Energy[2] (US Department of Energy 2011)	EU Commission[3] (European Commission 2014)	British Geological Survey (British Geological Survey 2015)
Year	2007	2008	2009	2011	2011	2014	2015
Elements Analyzed[4]	9 + PGM + 15 REE	64 + 5 REE	7 + PGM + 16 REE	12 + 2 REE	9 + 7 REE	51 + PGM + REE	36 + PGM +14 REE
Antimony							
Cadmium							
Cobalt							
Gallium							
Germanium							
Gold							
Indium							
Lithium							
Magnesium							
Manganese							
Molybdenum							
Nickel							
Niobium							
PGM							
REE							
Selenium							
Silver							
Tantalum							
Tellurium							
Tin							
Titanium							
Tungsten							
Vanadium							

[1] REE analyzed are dysprosium and neodymium

[2] REE analyzed are dysprosium, terbium, europium, neodymium, yttrium (High), cerium and lanthanum (Medium)

[3] REE divided in heavy and light. Heavy scored high risk, light medium risk.

[4] In some cases, economically important compounds are included

Table 1 shows the 23 most occurring elements in seven important studies developed between 2008 and 2015. As can be noticed, only two of these twenty-three elements appear in all seven the reports, namely gallium and indium. Moreover, the results of different reports often disagree with each other. Silver, for example, appears in four studies ranging from not critical to highly critical.

The reasons behind this type of disagreements are the different approaches and considerations adopted. If we compare the Oakdene Hollins 2008 and the EU Commission 2014 reports, a first look would suggest that the methodologies adopted are similar. In both cases a 2D matrix is built to evaluate the criticality of 50+ elements. A closer look, however, reveals one major difference: while the EU report considers Economic Importance along with supply risk, the Oakdene Hollins couples material risk to supply risk. In the first case, economic importance is obtained assessing the proportion of each metal associated with industrial megasectors at EU level. This is then combined with the gross value added by such megasector to the EU's gross domestic product (GDP). In the Oakdene Hollins work, instead, materials' use in different sectors is not considered. Material risk is a combination of environmental impact of the metal, substitutability, global consumption levels, etc. Given these two completely different approaches to assess criticality, we cannot expect to easily compare the results. Moreover, to further complicate such evaluation, also the common dimension, supply risk, cannot be directly compared. While the EU report includes substitutability as a major contributor to such dimension, the British Report, as illustrated earlier, includes this propriety in materials risk rather than in supply risk. As a consequence, fast and clear comparison between the two reports is not an easy task.

2.1.2 A Comprehensive Methodology for Determining Metal Criticality

In an effort to try and solve such non-easiness of comparison, Graedel et al. (Graedel et al. 2012) developed a comprehensive and flexible methodology for materials criticality determination. The proposed methodology is presented in a peer review paper titled "Methodology of Metal Criticality Determination" publish in 2012.

Graedel and colleagues implemented the original NRC concept, expanding the matrix by adding one more dimension. The result is a 3D matrix composed of supply risk (SR, same as the NRC report), environmental implication (EI, newly added dimension) and vulnerability to supply restriction (VSR, correspondent to the "Impact of Supply Restriction" by NRC). The methodology is developed at three organizational levels (corporate, national and global) in order to be fully applicable to any public and private company or institution.

Each dimension of the matrix is composed of two or more sub-groups (referred as components), which are themselves further decomposed in smaller entities (referred as indicators). Each indicator is assigned a score from 0 to 100. Moreover, each indicator and each component is assigned a weight, which is decided by the analysis. The result is a ramified structure with weighted indicators as building blocks which, through a group of appropriately weighted components, determine the overall score of that specific dimension, on a 0-100 scale.

Once all the different indicators have been detected and their values calculated, it is possible to determine the components value, through the chosen weighting system, and in cascade obtain the overall score of that specific dimension.
By combining the value of the three dimensions, an overall criticality score for the studied material is obtained. In this way the analysis is not only flexible, but also highly transparent. While the analyst is free to choose as many indicators as he believes are necessary and weight them as he feels more comfortable, it is very easy for the reader to detect the analyst's choices and judge whether the building assumptions make sense.

Besides introducing this new method, Graedel also provides guidelines concerning which components and indicators should be included in the assessment of metals' criticality according to his experience. The interested reader is invited to read (Graedel et al. 2012) and the relative supporting information for deepen explanation.

During the last few years, Graedel and colleagues applied the methodology described above to multiple elements of the periodic table. These analyses where reported in different publications (Harper, Diao, et al. 2015; Harper, Kavlak, et al. 2015; N T Nassar, Du, and Graedel 2015; Nedal T Nassar et al. 2012; Nuss et al. 2014) and a summary of these works was presented in a 2015 article titled “Criticality of metals and metalloids” (Graedel et al. 2015).

As underlined earlier, the methodology proposed by Graedel and presented above, it not intended to uniform all materials criticality related studies in order to obtain single, definitive results. Each study focus on different materials which are considered important for the economy of the interested company, region or country. The boundaries may differ greatly between two studies and the parameters considered too. Therefore, result should and are expected to differ between various studies. What Graedel and colleagues propose is a uniformed way to analyze available data and present results, maximizing transparency and facilitating the comparison of different studies. Moreover, in recent years Graedel focused on trying to better understand the so called “byproduct nature” of various metals. He refers to it as “companionality” and defines it “the degree to which a metal is obtained largely or entirely as a byproduct of one or more host metal from geologic ores” and locates it in the supply risk dimension of his methodology (N T Nassar, Graedel, and Harper 2015).

In the “Criticality of metals and metalloids” article it is found that “the metals of most concern tend to be those with three characteristics: they are available largely or entirely as byproducts, they are used in small quantities for highly specialized applications, and they possess no effective substitutes”.

The focus of this thesis is on the carrier-byproduct relation which is discussed in details in the following section.

2.2 Byproduct Metals

The carrier-byproduct relation is crucial when considering future low carbon energy scenarios, since most of the technologies involved use metals which are, de facto, byproducts (Fizaine 2013). Classic examples are cadmium, gallium, germanium, indium, selenium and tellurium, used in thin-film photovoltaics cells, platinum, palladium and the other PGM essential in fuel cells, among other applications (Andersson 2000). Carrier-byproduct linkages for copper-indium-gallium-selenide photovoltaic cells are shown in Figure 2, where the main carriers for each byproduct metal are represented by the upper circles.

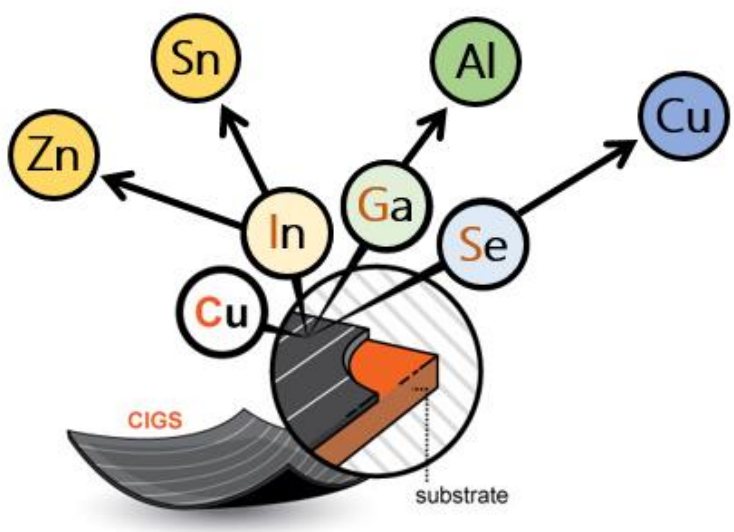


Figure 2: Carrier-byproduct linkages in CIGS PV cells (adapted from (Endless Solar Sun 2016)*)*

But the byproduct constraint is not limited to metals used in clean energy technologies. Many other metals are mined as byproduct and this affects various sectors. Antimony, bismuth, cerium, cobalt, europium, hafnium, lanthanum, molybdenum, praseodymium, rhenium, silver, tantalum and zirconium are some of the metals mined mainly as companions (N T Nassar, Graedel, and Harper 2015).

The main advantages of producing metals as byproducts are the lower extraction and processing costs, as those are split between both the companion and the host. An example is copper-molybdenum recovery from copper tailings at Amerigo Resources' operation "El Teniente" in Chile. The Canadian company, despite a period of low price for both Cu and Mo, managed to increase its revenues by processing old and new copper tailings, rich in the two elements (Fraser 2012).

On the other end, from a resource availability point of view, the carrier-byproduct relationship has one main implication: the maximum supply potential of the byproduct is limited by the extraction of the hosts. This limitations has led to criticality concerns for byproduct metals (Graedel et al. 2015).

As it can be seen from Table 1, most of the cited byproduct metals always or almost end up classified as medium-to-high criticality elements. Gallium, indium and tellurium, for example, are found highly critical at least 50% of the times. Cadmium, germanium and silver are label "high criticality" at least one time and selenium falls in medium criticality region three out of four times.

One of key characteristics of byproduct metals pointed out in criticality related studies is their high price volatility. According to Graedel et al., in most cases price fluctuations of companions metals are a direct consequence of the carrier-byproduct relationships, which cause prices to be inelastic in response to their supply. In other words, demand of a certain byproduct metal rises, but supply cannot catch up since its limited by the extraction of the host metal. For the mining company, in fact, the revenue from selling the companion metal is not sufficient to cover the increased cost of extraction. An increase in byproduct's price follows since the supply is unable to match the growing demand. Detailed explanation of this phenomenon, with some background and its implications, is provided in the methodology section.

The carrier-byproduct linkage has both positive and negative impacts, as discussed above. On one side production costs can be minimized by combining the extraction and processing of different metals at one single facility. On the other hand, supply of the host metal may limit supply of the byproduct. In addition to this, most of the metals mined as byproduct are classified as "minor metals”. Although no universal definition is available, according to Meskers and Hagelüken (Meskers and Hagelüken 2010) minor metals can be defined as “[they are] metals that have relatively low production or usage, which occur in low ore concentrations, are regarded as rare, or are not traded at major public exchanges”. Such definition contains various potential sources of criticality. For example, by not being traded on major metal markets, such as the London Metal Exchange, prices may not be driven by the normal market forces. The limited number of either suppliers or consumers may indeed have high influence on price (Redlinger and Eggert 2015).

In order to minimize criticality issues related to the ”minor metal” nature, increased recycling and materials substitution should be considered (N T Nassar, Graedel, and Harper 2015). For what concerns recyclability, a 2011 report by the International Resource Panel, shows how most of these minor metals have very low end-of-life recycling rates, with gallium, germanium, indium, selenium, tellurium and the REE all being below 1% (Graedel et al. 2011). There is therefore large space for improvements. Nevertheless, without incentives driven by strong recycling policies, this is not expected to occur soon. The costs of sorting electronic components, such as PV cells and smartphones, are in fact very high.
Substitution of materials also could strongly contribute to the reduction of metal criticality. As shown in the “Complex Life Cycles of Precious and Special Metals” study by Meskers and Hagelüken (Meskers and Hagelüken 2010), depending on the field of application, each minor metal typically has at least one substitute materials which allow the technology to still perform well. However, as pointed out later in the article, in many cases the optimal substitute element is itself a minor metal and the criticality issue is simply shifted from one metal to another. The most classical example is platinum and palladium, which due to their similar chemical and physical proprieties can substitute each other in many applications. However, their similarities cause them to occur in the same deposits, therefore making them both subjective to similar geological issues.

In conclusion, although the carrier-byproduct linkage has been studied by many scholars and institutions worldwide, there seems to be little understanding of the phenomena and its implications. Graedel states that “One aspect of companionality of note is that when a metal is obtained largely or completely as a companion, its production is often unable to respond quickly to rapid changes in demand and, as a result, its price can fluctuate widely", but report only rhenium as one, single example (N T Nassar, Graedel, and Harper 2015). Moreover, the British Geological Survey do not analyze tellurium in the “2015 Risk List” given its byproduct nature and the related difficulty to find reliable data.

In order to better understand the implication of the byproduct relation in the determination of its criticality, better data collection is needed coupled with a broader analysis of available data. As PV sector is one of the main users of many byproduct metals, its market and related materials are introduced in the following section and later used as case study in order to evaluate price elasticity of indium and tellurium.

2.3 PV Metals

Every hour, the amount of solar energy absorbed by earth's atmosphere, oceans and lands, is equivalent to humanity's primary energy demand for one entire year (United Nations Development Programme 2000). Given this, it's not surprising that solar energy is considered one of the pillars of clean energy revolution. Solar energy is indicated as one of the main energy sources of the future in every global energy forecasting report, such the "World Energy Outlook" and the "Energy and Climate Changes" by the International Energy Agency (IEA).

Solar energy can be directly harnessed in three forms: as heat, as electricity or as liquid fuels, like hydrogen. Out of the three methods, direct electricity generation, achieved through photovoltaics cells, is the most promising one at this stage. The simplicity of the systems involved, the scalability and the large scale manufacturing of different types of PV cells, made grid parity possible for solar energy starting from 2013. This means that electricity generated by photovoltaics systems is cost competitive with traditional sources, such as hydro, coal, gas turbines and nuclear power plants, even without incentives. At first, it was Spain and some remote islands, now more than 30 countries have reached PV grid parity, at least in some regions (Shah and Booream-Phelps 2015).

In terms of capacity, in the last two decades PV cumulative installed capacity increased 100-folds, reaching 230 gigawatts peak in 2015 (BP plc 2016). One main technology is the core of this exponential growth: crystalline silicon (c-Si) cells. As of today, crystalline silicon cells accounts for more than 90% of world PV cells production (Fraunhofer Institute for Solar Energy Systems 2016) and the high market share is a consequence of a good trade-off between cost and performances, as well as of the maturity of the technology.

Cells are classified as either monocrystalline or multi-crystalline (polycrystalline), depending on the crystallization present. The main differences between the two types are production costs and efficiency. While monocrystalline silicon wafers are more expensive to manufacture, they can achieve higher conversion efficiency. According to the National Renewable Energy Laboratory the top laboratory efficiency is 21,3% for multi-crystalline cells and 25,6% for monocrystalline ones (National Renewable Energy Laboratory n.d.).

Crystalline silicon solar cells, however, do not perform well from an environmental point of view. The production of high purity monocrystalline silicon is an energy intense process, requiring around 300 kWh/kg for the conventional Siemens c process followed by Czochralski growth of the crystal (Pizzini, Acciarri, and Binetti 2005). The result is an energy payback time of several years, depending on the location where the PV is installed. In other words, it takes few years of operation of the PV system in order to generate as much energy as the one used during manufacturing. But the consequences are not limited to energy payback time: high emissions of greenhouse gases, criteria pollutants and heavy metals are also present during crystalline silicon wafer production (Fthenakis et al. 2011). In an attempt to minimize environmental issues, less energy intensive processes have been developed, partly switching to multi-crystalline silicon cells. Moreover, the amount of silicon per unit area has been reduce by a factor of two from 1990, decreasing the wafer thickness from 400 to 200 µm (Fraunhofer Institute for Solar Energy Systems 2016). The objectives have been achieved, reducing both energy payback time and emissions. Nevertheless, crystalline silicon PV cells are not as environmental-friendly as other types of cells.

In order to further minimize costs and cut emissions, different types of solar cells started to be developed in parallel with crystalline silicon cells. The most promising type, which now account for about 10% of global market, are thin-film cells. Copper-indium-gallium-selenide, copper-telluride and amorphous silicon (a-Si) cells all belong to this class, with the first two dominating the market (Fraunhofer Institute for Solar Energy Systems 2016). As the name suggests, thin-film PV make use of thinner layer of semiconductor materials, ranging from few nanometers to tens of micrometers. The advantages are multiple: flexibility, low weight, lower environmental impact and reduced costs, since the semiconductor materials can be deposited through inexpensive vapor phase techniques. For some time, the lower efficiencies compared to c-Si cells have been the main bottleneck for the establishment of this technologies. In recent years, however, thanks to the interest and efforts of large companies such as First Solar, efficiencies of thin-film PVs rose significantly, overtaking in some cases multi-crystalline silicon cells. At today, top cell laboratory efficiencies are 22.3%, 22.1% and 13.6% for CIGS, CdTe and (a-Si) respectively (National Renewable Energy Laboratory n.d.) and this allow thin-film PV to survive in an era where c-Si cell have reached record low production costs.

The reduced use of materials, besides being one of the main advantages of thin-films cells, is also one of the causes of major concern. Silicon is substituted with III-V or II-VI semiconductor compounds, such as gallium-arsenide and cadmium-telluride, lowering cost, weight and environmental implications, but raising metal criticality concerns. Most of the metals used in CIGS and CdTe cells, in fact, are minor metals, often produced as byproducts. Cadmium, gallium, indium, tellurium, selenium are all example of this trend. The problem is the huge quantity required of this materials, which is in the order of dozens kg per MWp of installation. In 2010 report by USGS, the amount of byproduct metals required for the generation of 8760 gigawatt hours of energy are presented. This amount of energy is equivalent to four gigawatts of installed peak power, considering a capacity factor of 25%. This last figure accounts for daylight hours and other factors affecting electricity production. The quantities required, together with the percentage of 2008 world refinery production from primary sources are summarized in Table 2.

Table 2: Byproduct metals quantities required in CIGS and CdTe PV cells for certain energy requirements

Technology	Metal	Quantity (tons)	Primary production percentage	Quantity (kg/MWp)
CIGS	Gallium	30	27	7.5
	Indium	90	16	22.5
	Selenium	180	6	45
Cd-Te	Cadmium	340	2	85
	Tellurium	390	82	97.5

The last column represents the quantities in kilograms per megawatt peak. As it can be seen, large amounts of byproduct metals are required both in CdTe and CIGS cells. For example, cadmium telluride cells use almost 100 kg of tellurium for MWp of power. Considering todays' global Te supply, which is estimated to be around 500 tons per year (Kavlak and Graedel 2013b), the maximum potential annual installation of CdTe solar cell is around 5GWp. Given an actual installation of 1.9GWp per year (Fraunhofer Institute for Solar Energy Systems 2016), CdTe cells are already very close to their maximum development.

Byproduct metals issues, however, are not limited to thin-film cells. Tin and silver, for example, are used in the production of c-Si modules. According to the 2011 "Critical Metals for Strategic Energy Technologies" report by the Joint Research Centre of the EU, 24 and 577 kg of the two byproduct metals respectively are required per megawatt peak installed.

It is therefore essential to understand the characteristics and limitations of the metals involved in the production of solar cells, especially focusing on those materials mined as byproducts. If one hand, cells' improvements are expected in the coming decades, such as further reduction of materials use and increase of efficiencies, significant increase of annual installed capacity is expected to occur soon. In order to reach high PV energy penetration, such as the 16% forecasted by IEA in the high renewable scenarios for 2050 (International Energy Agency-IEA 2014), many gigawatts of new photovoltaics system are in fact required.

3 Methodology

As outlined in the introduction, the scope of this work is to assess the impact of byproduct mining as indicator of materials criticality, in particular by looking at price elasticity of PV related metals.

The thesis is divided in two main parts.

The first goal of the work consists of the data collection for different carrier-byproduct pairs across the periodic table and development of a 2D criticality matrix. This includes byproduct fraction on one dimension and value ratio on the other one, with the first reflecting the amount of primary production obtained as byproduct and the second representing how valuable is the studied byproduct for the mining company. Detailed explanation of the collected data, their sources and implications are explained in chapter four.
It is expected that the different pairs behave in a certain way from a market point of view, based on their byproduct fraction and value ratio. The matrix is analyzed both qualitatively and quantitatively, using hierarchical clustering for the quantitative part. Some results are shown elementwise, therefore considering the contribution from multiple carriers. A simple, semi-quantitative evaluation of uncertainty is carried out. The purpose is to use the outcomes to test the robustness of the quantitative clustering analysis, rather than providing accurate uncertainty ranges. Its brief description is therefore skipped at this point and included in chapter 4, after the description of the data.
It is decided to collected the data per pair and not per metal, as often done in literature. This choice is driven by the necessity of understanding the potential of recovery of the byproduct based on all the different carriers, by looking at each one of them, one by one. In this way, the outcome of this work is expected to be useful for metals' suppliers. By increasing the awareness concerning the raw materials they process they might be encouraged to increase recovery of byproduct metals, therefore minimizing possible supply shortages.

The second part of this work consists of the analysis of one specific cluster. Tellurium as by product of copper and Indium as byproduct of zinc are chosen as representative of the cluster showing high byproduct fraction. As explained in the 'Byproduct Metals' background chapter, large byproduct fraction has been cited as a potential source of metal criticality, given that the maximum supply of the companion may be limited by its carriers. In particular, price inelasticity of the byproduct metal has been used to justify such constraint.

3.1 Supply and Demand Model

In order to analyze price elasticity of byproduct metals, an introduction of classic supply and demand theory is required.

Demand is defined as the quantity of a certain product or service buyers are willing to buy, at a certain price. Supply, on the other hand, represent the quantity of a product or service producers are willing to sell, at a given price. According to supply and demand theory, in a perfect competitive market the price of a good or service will vary until demand and supply quantities are in equilibrium. Perfect competition is possible when buyers have complete information about the product they are buying, there are no barriers to enter or exit the market, products sold are homogeneous and every participant is a price taker, meaning that there is no supplier or buyer large enough to set prices.

The equilibrium character of a competitive market can be explained by analyzing supply and demand separately.
The law of demand states that, everything else being constant, the higher the price, the less the demand. The law of supply, instead, behaves conversely. The higher the price of a certain good or service, the more producers will be interested in selling it. Both behaviors are shown in Figure 3, with A referring to demand and B to supply.

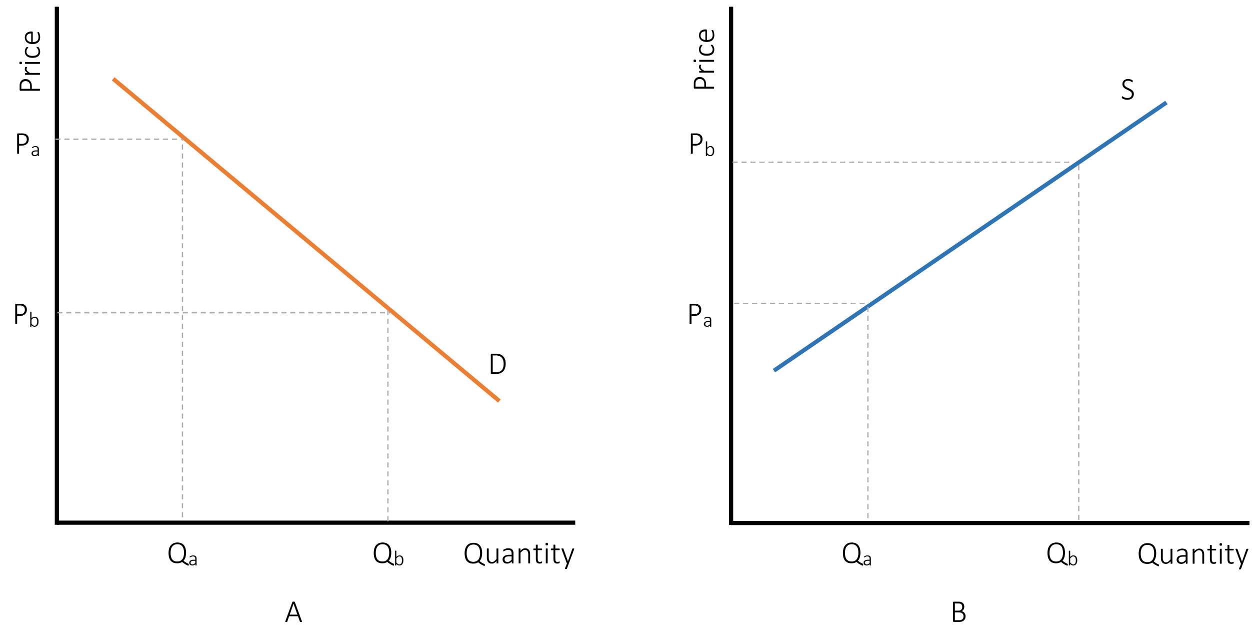


Figure 3: Demand and supply curves. Fig. A represents the demand curve, while Fig. B the supply one.

Given the opposite trends, there must exist a certain price for which supply equal demand. When this happens, equilibrium is reached and the economy is most efficient. At that particular price, the number of goods supplied matches perfectly the amount demanded, and both producers and consumers are satisfied. Equilibrium is shown as the intersection of the supply and demand curves in Figure 4.

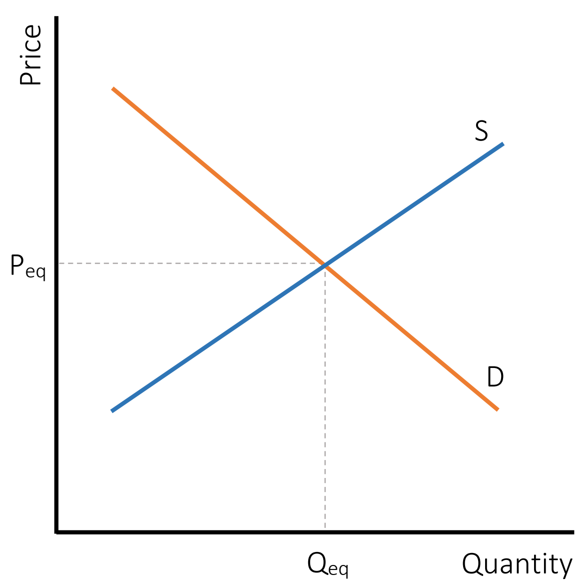


Figure 4: Supply and demand equilibrium represented by the intersection of two curves

For what concerns elasticity, a good or service is said to be price elastic when the quantity sold or demanded is responsive to changes in price. Therefore, a certain good is price elastic to supply if a decrease of price is followed by a decrease of supply. Similarly, the good is price elastic to demand if a rise of price is followed by a decrease of demand. If supply or demand does not respond to changes in price, it is said to be price inelastic. Although there is no clear definition of the boundary between elastic and inelastic regimes, Ivan Png Paal Liang (Png Paal Liang 1999) defines supply elastic goods having a coefficient equal or greater than one, and demand elastic those with coefficient smaller or equal to minus one . Clearly, when supply or demand are perfectly inelastic, the coefficient is zero, since a change in price does not affect quantities produced or sold.
Moreover, it is well-known that steel is price inelastic and the demand elasticity is in the range of -0.2 to -0.3 according to (Zhu 2012).

3.1.1 Supply and Demand Model for Byproduct Metals and Price Elasticity

The concept of price elasticity in the contest of byproduct metals has been analyzed in a 2015 peer review paper titled "Volatility of byproduct metal and mineral prices" (Redlinger and Eggert 2015) and is presented here.

Supply and demand curve for goods that are produced as byproducts, such as metals, are more complicated that the basic case shown above. Figure 5 is adapted from (Redlinger and Eggert 2015) and illustrates supply and demand curves for a certain metal.

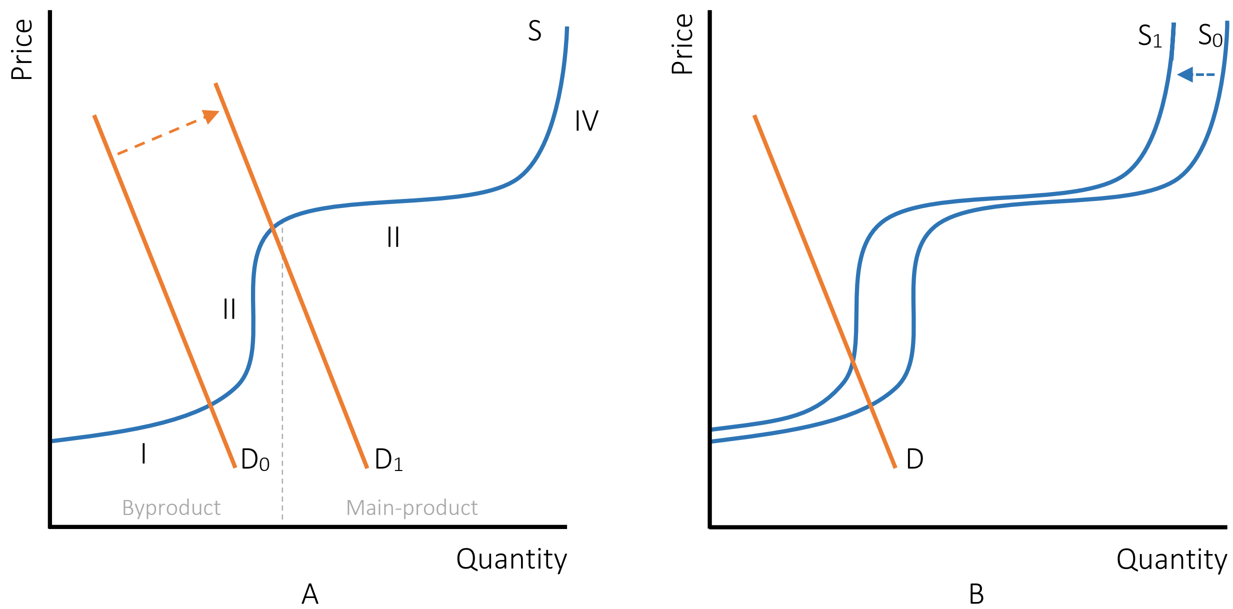


Figure 5: Supply and demand curves for byproduct metals. A illustrates the effect of an increase in demand of the byproduct, while B the effects of a decrease in supply of the carrier

Again, the blue lines represent supply of the metal, while the orange ones the demand. According to the nature of metal, i.e. byproduct or main product, different parts of the supply line are of interest. Four parts can be distinguished, has seen in Figure 5.A. Part I reflects a byproduct in the elastic regime: its supply is not yet limited by the carrier and therefore an increase in demand follows an increase in price. In part II the byproduct has almost reached its maximum supply potential limited by the carrier. An increase in demand, as shown I Figure 5.A, cannot be matched by supply, therefore causing the price to rise sharply. At this point, quantities produced are non-responsive to changes in price and the byproduct is in the inelastic regime. If high demand of the byproduct metal persists, the mining company will invest in new machineries and increase its capacity and eventually the byproduct turns to be the main product of that specific operation. Costs of extraction and processing rise together with the price of the metal. This is represented by the large jump in the supply curve between part I and III. From there on, the metal is mined as a main product and the regime is again elastic (III). In situation IV, capacity of the main metal is reached due to either excess of demand or degradation of the ores, new investments are needed, costs of extraction increased and price peaks.

In Figure 5.B, a negative supply shift of the carrier metal is reflected on the byproduct, which suffers a supply shift even though its demand remains unchanged. As a consequence, demand is not met and the result is an increase of the price, which allocate the byproduct in the inelastic regime. In short, Figure 5.A represents a byproduct metal entering the inelastic regime due to increase in its demand. On the hand, Figure 5.B shows a byproduct metal entering the inelastic regime due to decrease in carrier supply.

In order to verify whether the supply of the host metals causes the inelastic behavior of the byproduct, Tellurium and Indium supply elasticities are analyzed over a certain time period. Due to lacking of reliable demand data, ideal market is assumed. As a consequence, supply and demand quantities are in equilibrium and are referred simply as quantity.

According to economic theories, quantities can be expressed in terms of price as the following equation shows.

$$Q = \varphi P^{\alpha} \tag{3.1.1.1}$$

When log terms of both quantity and price are used, a linear relation is found, where elasticity is described by the coefficient α.

$$\log Q = \alpha \log P + log\varphi \tag{3.1.1.2}$$

Quantity, does not depend solely on price and therefore other terms are needed in order the describe it. The general structural equation is proposed in 3.1.1.3, where Q and P are already expressed as log.

$$Q_t = \alpha_1 P_t + \alpha_2' W_t + \varepsilon_t \tag{3.1.1.3}$$

where Q express quantity (supply or demand), P is price, W a set of variables (called shifters), t indicates time and ε is the shock term. The prime on the α_2 coefficient represents the transpose: W and α_2 are in fact vectors, represented by multiple variables, and therefore transpose of α_2 is needed when dealing with their product.

3.2 Econometric Analysis for Price Elasticity

Price and quantity time series are collected for both indium and tellurium, as well as other variables needed for the models. Econometric analysis is performed. In particular, three linear regression models are developed and the results compared. All quantity and price variables are log transformed. This is done in order to obtain linear models and also to reduce the variability of the chosen variables.

3.2.1 Ordinary Least Square (OLS) Model

The first model is ordinary least squares.
A common practice by econometricians, is to model supply and demand elasticity at the same time. In what follows, a model is developed based on the work of Wright on supply and demand for tobacco (Stock and Watson 2003). Supply and demand equations of the OLS model are shown below, where 3.1.1.4 is the supply equation and 3.1.1.5 the demand one.

$$Q_t^s = \alpha_1 P_t + \alpha_2' Z_t^s + \alpha_3' W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp P_t, Z_t^s, Z_t^d, W_t \tag{3.2.1.1}$$

$$Q_t^d = \beta_1 P_t + \beta_2' Z_t^d + \beta_3' W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp P_t, Z_t^d, Z_t^s, W_t \tag{3.2.1.2}$$

Exogeneity of the independent variables is shown on the right, using the $a \perp b$ symbol which is short for *corr(a,b)=0*. Exogenous variables are variable outside the model and are not explained by variables in the model. The independent variables are split in Z and W terms and α_i substituted with α'_1 and α'_2, with the prime implying that multiple variable and therefore coefficient are included. The constant term is included in W.

While W terms are common shifters, i.e. influence both supply and demand, the Z ones are specific either for the supply or demand equation. A supply shifter shifts the supply curve in ways that are uncorrelated to demand; demand shifters are the exact opposite. The function of supply shifters is illustrated in Figure 6.

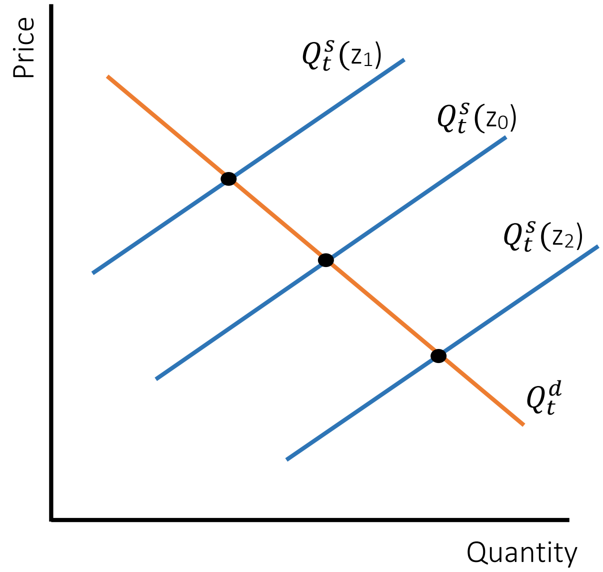


Figure 6: Illustration of the effects of supply shifters

Supply shifters cause the supply curve to shift in ways that are uncorrelated to the demand. The different equilibrium points can be used to identify the demand curve and thus find the demand elasticity coefficient. Similarly, demand shifters allow identifying supply elasticity.

The OLS model is based on the following assumptions:

- Strict Exogeneity: the regressors are uncorrelated with the error term, which mathematically is expressed as $\varepsilon_t \perp P_t, Z_t, W_t$;
- Homoscedasticity of error terms: variance of the error term is constant;
- No autocorrelation: error terms are uncorrelated with their own lag terms[5]

Due to the nature of variables used in the model, the assumptions above might not always be true. For what concerns homoscedasticity of the error terms, this can be fixed using the White estimator (White 1980). It consists of calculating the standard deviation in a slightly different way, therefore dealing with small sample issues. In order to deal with strict exogeneity and serial autocorrelation, two other models are instead needed.

Price elasticity is represented by the coefficients α_1 and β_1 in Equation 3.2.1.1 and 3.2.1.2.

[5] For a time series data $\{Y_t\}_{t=m}^{T}$, the *n-th* order lag terms are $\{Y_{t-n}\}_{t=m}^{T}$

3.2.2 Autoregressive Distributed Lag Model (ARDL)

In cases where the error terms exhibit autocorrelation, lag terms of variables should be included in the OLS model to deals with this problem.

The OLS model serves as starting point for this model. Ideally, the number of variables should be in the range $n/20 \sim n/10$ (Harrell 1986). In order to limit the number of variables, stepwise regression is used. Price is always forced into the model, since price elasticity of supply is the scope of this analysis. Stepwise regression is a procedure able to identify which variables should be included in the model, based on a certain criterion. Stepwise regression can be forward, backward or bidirectional. Forward starts with no variables in the model, which are later added one by one. Each time one variable is added, the criterion is evaluated and the regression goes on until no more improvements occur. Backward selection works in the opposite way: the model starts with all the variables and one by one they are removed, until no more improvements are possible. Bidirectional consists of a combination of the two methods, one variable can be added and removed at each step. All three stepwise regressions are developed, based on the Bayesian Information Criterion (BIC). BIC is a goodness of fit indicator which includes a penalty term for the complexity (too many independent variables for too few observations) of the model.

At this point, residual of the stepwise regression should be assessed to understand if there is autocorrelation. Then, vector autoregressive model (VAR) is used to determine how many lag orders to include in the model (Hamilton 1994). It should be noted that VAR does not provide any information about which variable's lag term to include, but simply suggest if first, second, third or other orders should be used.

From then on, n lag terms of each m variable are either included or excluded to produce m^{n+1} possible combinations and the adjusted R^2 are determined for each case. The regression showing the greater adjusted R^2 is chosen to represent the Autoregressive Distributed Lag Model and supply elasticity is determined from it.

The two equations, one for supply and one for demand, are presented here.

$$Q_t^s = \sum_{q=1}^{n_q} \alpha_q Q^s{}_{t-k} + \sum_{p=0}^{n_p} \alpha_{1,k} P_{t-p} + \sum_{z=0}^{n_z} \alpha'{}_{2,z} Z^s{}_{t-z} + \sum_{w=0}^{n_w} \alpha'{}_{3,w} W_{t-w} + \varepsilon_t^s \,, \qquad \varepsilon_t^s \perp P_t, Z_t^s, Z_t^d, W_t \qquad (3.2.2.1)$$

$$Q_t^d = \sum_{q=1}^{n_q} \beta_q Q^d{}_{t-q} + \sum_{p=0}^{n_p} \beta_{1,p} P_{t-p} + \sum_{z=0}^{n_z} \beta'{}_{2,z} Z^d{}_{t-z} + \sum_{w=0}^{n_w} \beta'{}_{3,w} W_{t-w} + \varepsilon_t^d \,, \qquad \varepsilon_t^d \perp P_t, Z_t^d, Z_t^s, W_t \qquad (3.2.2.2)$$

where *q*, *p*, *z* and *w* represent the lag of the correspondent variables and n_q, n_n, n_z and n_w the maximum order of lag. While for *p*, *z* and *w* the lead term (original time series) is included, this is not the case for *q* as the lead term of quantity is already present on the left side of Equation 3.2.2.1 and 3.2.2.2.

In the ARDL model lag terms are inserted in order to deal with autocorrelation. As a consequence, two different types of elasticities are present. The first, referred as short run, represents the immediate effect of a unit change in price and therefore do not considers this effect to be projected in the future. Long run, instead, deals with such projection and is used when log terms are introduced.

Consider the simple model: $$Q_t = c + \gamma Q_{t-1} + \alpha P_t + \varepsilon \qquad (3.2.2.3)$$

where Q_{t-1} represents the first lag of quantity. The instantaneous effect of *P* onto *Q* is measured by α (short-run elasticity). Since P_t has an effect on Q_t, P_t will also have an effect on Q_{t+1} through the lagged dependent variable and the size of the effect is measured by $\gamma\alpha P_t$. The effect is not limited to the first lag. Through second lag, P_t will have an effect also on Q_{t+2}, measured by $\gamma^2\alpha P_t$. And so on. The cumulative effect, denoted as long-run elasticity, can be shown to be

$$\alpha + \gamma\alpha + \gamma^2\alpha + \ldots = \frac{\alpha}{(1-\gamma)} \qquad (3.2.2.4)$$

According to economic theories, long-run elasticity is always greater than short-run as γ is smaller than one. When lag terms are added to the model, it is therefore not sufficient to consider short-run elasticity in order to verify supply inelasticity. In ARDL, supply elasticity in response to price is therefore calculated according to Equation 3.2.2.3, where γ represents the coefficient of the lagged terms of quantity.

3.2.3 Instrumental Variables Model (IV)

The first assumption of OLS model is that all regressors are exogenous. In supply and demand models, when predicting the quantity supplied in equilibrium, price is known not to be an exogenous variable. Changes in quantity leads to changes in price and vice versa and therefore price is referred as endogenous; $\varepsilon_t \perp P_t$ is not valid. Theoretically, the first assumption of the OLS model is therefore not satisfied. To deal with this issue, an Instrumental variable model is developed. IV models manage to provide a consistent estimate by using instrumental variables, which by definition must be correlated with the endogenous variables, but not correlated with the error term.
Two Stages Least Squares (2SLS) technique is chosen in order to run the model. As the name suggests, 2SLS consists of two stages of regression. This can be summarized in the following set of equations, with the first four equations referring to supply and the other four to demand.

$$Q_t^s = aP_t + a_2'Z_t^s + a_3'W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp Z_t^s, Z_t^d, W_t;\ corr(P_t, \varepsilon_t^d) \neq 0 \tag{3.2.3.1}$$

$$P_t = \gamma_1'Z_t^d + \gamma_2'W_t + \varepsilon_t \qquad \varepsilon_t \perp Z_t^d, Z_t^s, W_t; \tag{3.2.3.2}$$

$$\hat{P}_t = \gamma_1'Z_t^d + \gamma_2'W_t \tag{3.2.3.3}$$

$$Q_t^s = \alpha_1\hat{P}_t + \alpha_2'Z_t^s + \alpha_3'W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp Z_t^s, Z_t^d, W_t \tag{3.2.3.4}$$

$$Q_t^d = bP_t + b'Z_t^d + b'W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp Z_t^d, Z_t^s, W_t;\ corr(P_t, \varepsilon_t^d) \neq 0 \tag{3.2.3.5}$$

$$P_t = \delta_1'Z_t^s + \delta_2'W_t + \varepsilon_t \qquad \varepsilon_t \perp Z_t^s, Z_t^d, W_t \tag{3.2.3.6}$$

$$\hat{P}_t = \delta_1'Z_t^s + \delta_2'W_t \tag{3.2.3.7}$$

$$Q_t^d = \beta_1\hat{P}_t + \beta_2'Z_t^d + \beta_3'W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp Z_t^d, Z_t^s, W_t \tag{3.2.3.8}$$

where equations 3.2.3.1 and 3.2.3.5 represent the starting point-OLS or ARDL model-and may therefore contain lag terms which are implicitly included in Z and W. Endogeneity of price is mathematically represented by $corr(P_t, \varepsilon_t^{s/d}) \neq 0$.

First, each endogenous variable (in this case only price) is regressed over a set of instruments, as shown in 3.2.3.2 and 3.2.3.6. In order to deal with the exogeneity problem, the instruments must be uncorrelated with the error term. A common choice suggested in literature is to use demand shifters in the supply equation and supply shifters in the demand equation (Zhu 2012). By doing so, estimators of the endogenous variables are obtained, indicated as $\hat{P}_t$ as shown in equations 3.2.3.3 and 3.2.3.7. Secondly, an OLS regression is performed on the complete model, with the only difference that the endogenous variables are replaced by the estimators obtained in the first step of 2SLS. This second step is presented in Equations 3.2.3.4 and 3.2.3.8. The endogeneity problem is therefore eliminated and price elasticity can be consistently estimated using equation 3.2.2.3.

The validity of the IV model should be compared to the autoregressive distributed lag one. According to (Greene 1993), three statistical test should be run. The first two are used to test the goodness of the chosen instruments, while the third compares the consistency of the estimators obtained with the ADRL and IV models.

As seen earlier, a good instrument should fulfill two requirements: be correlated with the endogenous variable and show no correlation with the error term. An F test and the Sargan-Hansen test are performed to verify each requirement.
Correlation with the endogenous variable can be verified using the F test, with price ad dependent variable and the instruments as independent ones.
On the other hand, if the number of instruments is greater than the number of endogenous variables, the Sargan-Hansen test for over identifying restrictions can be used to verify non-correlation of instruments with the error term. Under the null hypothesis, exogenous variables are indeed exogenous.

One last test, the Wu-Hausman test, is needed to verify the consistency of the IV estimator. Under the null hypothesis, the autoregressive estimator is as consistent as the IV one. In other words, if the null hypothesis is verified, the use of IV model is not justified. There can be two explanations why this happens: either the chosen instruments are not good, or the endogenous variable is indeed exogenous. If goodness of instruments was verified with the first two tests, then non-endogeneity of the variables is the cause of IV estimator being as consistent as the ARDL one. There is no statistical evidence that price is endogenous.

4 Categorizing Carrier-Byproduct Pairs

The first goal of this work is the categorization of different carrier-byproduct metals in a 2D matrix. Once the matrix is built, qualitative and quantitative analysis is performed, with the purpose of identifying pairs which have similar market behaviors and study them separately.

4.1 Data

4.1.1 Data Sources

In order to build the matrix, byproduct fraction and value ratio are needed. While the first is straightforward, the second is slightly more complex, deriving from multiple sets of data.
Byproduct fraction is defined as the fraction of overall primary production (derived from ores) which is obtained as a byproduct from the carrier.
Value ratio, instead, represents how valuable is the studied byproduct for the mining company and is calculated as follows.

Value ratio = Quantity ratio · Price ratio = (Concentration ratio · Efficiency ratio) · Price ratio

Concentration represent the concentration of the metal in the ores, while efficiency reflects how much of the available metal is recovered.
All the ratios data refer to minor metal over carrier.

The data sources used are described as follows:

Government Scientific Agencies

- United State Geological Survey (USGS)
- British Geological Survey (BGS)

Materials Encyclopedias

- Ullman's Encyclopedia of Industrial Chemistry (Kirk et al. 2004)
- Kirk-Othmer Encyclopedia of Chemical Technologies (Ullmann's Encyclopedia of Industrial Chemistry 2011)

Material Institutes

- The Silver Institute
- Tantalum-Niobium International Study Centre
- Cobalt Development Institute

Companies Annual Reports

Journal Articles

Commodities Exchange

- London Metal Exchange (LME)

4.1.2 Data Selection

Byproduct fraction and concentration ratio data were found for most of the pairs identified in literature, but this is not the case for efficiency ratio. Recovery rates, in fact, are usually not included in companies' annual reports and therefore are not accessible to the public. It is decided to assign efficiency ratio of one, when the data is not available. This approximation is justified by the fact that quantity ratio is more strongly affected by concentration ratio, rather than efficiency ratio. While concentration ratio sometimes differs by one or more order of magnitude between different sources, efficiency ratio is always of the same order, mostly equal or greater than 80%

The data of tellurium as byproduct of copper are reported here in order to provide insight on how data are collected. For explanation of the various columns, such as "Year of determination", "Number of mines", etc., see the 'Uncertainty Analysis' section.

Table 3: Data collection example - tellurium as byproduct of copper

Byproduct fraction			Concentration ratio				Efficiency ratio			
Value	Year[6]	Level of approximation	Value	Year	Number of mines	Sources number	Value	Year	Number of mines	Sources number
<85%	2014	Reported *number*	$2.5 \cdot 10^{-4}$	2012	Global Average	1	0.33	1985	Worldwide	1
100%	2010	"Nearly all"	$1.1 \cdot 10^{-4}$	2003	Global Survey (63 refineries)	1	0.55	2011	Worldwide	1
>90%	2016	"More than 90%"	$2 \cdot 10^{-4}$	1985	Worldwide	1	0.42	1973	Worldwide	1

As it can be seen in Table 3, multiple data per category are usually available for each pair. In order to obtain one single data (one for Byproduct fraction, one for concentration ratio, one for efficiency ratio) to move forward for the grouping stage, one intuitive approach could be to use the average of the available values. However, in many cases this doesn't seem appropriate, due to reliability of data.

As a general guide, more recent and "level-wide" (global level instead of one single mine) data are preferred. In particular, most recent data are usually chosen for byproduct fraction and efficiency ratio. In many cases, in fact, due to increased demand of a particular metal or thanks to technological improvements, it's possible to separate the metal from the carrier with a higher recovery rate. The result is an increase of efficiency rate and a change in the share of primary production of that specific metal (change of byproduct fraction from the various carriers). For what concerns concentration ratio, instead, it is considered more important the level of the examined study (sample size) rather than the time at which the concentrations were measured. It is expected that concentration do not vary largely in time, while they can differ by order of magnitude between different geographical areas. Therefore, a study considering ores from a great number of refineries all

[6] Year stands for Year of determination

around the world is preferred to a study focusing only on the resources of one specific country or mine. It should be noted that these are general guidelines and each value is evaluated case by case. Cu-Te, again, is a great example. The byproduct fraction, in fact, was obtained combining the widely accepted "90% of primary production is from copper anode slimes", with the recent information provided by USGS in 2016 which states that 15% of world tellurium production is now obtained from mines where Te is the main product. This was confirmed by checking mining companies reports and websites, where this new projects were described as primary tellurium mines. As a consequence, a byproduct fraction of 76% was chosen, which correspond to 90% of the 85% now mined as byproduct.

When multiple data seem to have similar overall reliability, a mean value is used. The chosen values for each pair are reported in the Appendix.

Complete data are found for 47 out of the 70 identified pairs. Most of the remaining 23 pairs are cited in literature, but the quantity and value of the minor metals are so small that they do not end up reported by the producers. Moreover, from a global perspective, the quantity obtained from that carrier is irrelevant and therefore no studies have been focusing on the estimation of concentration and fraction of the byproduct.

4.2 Uncertainty Analysis

Best efforts have been made in order develop a relatively comprehensive statistical uncertainty analysis, given the nature of available data. In many cases, in fact, collected data are either upscaled from one or few mines, companies or countries, or estimated based on surveys, historical trends or industry expert opinions. Nevertheless, it is decided to semi-quantitatively analyze uncertainty. Specific information that may affect reliability is collected together with the data, as shown in Table 3. This includes year of determination, number of sources or calculations involved, number of agreeing sources, etc.

The collected information is used to evaluate uncertainty. This should be considered a first attempt to determine uncertainty and could be a starting point for scholars interested in the field of uncertainty analysis. It is therefore decided to refer at the result of such analysis as semi-quantitative.

In order to simplify the calculations, price data are considered to have negligible errors. The high accuracy at which price is reported (especially true for those metal traded on LME and other open markets) is considered sufficient to justify such choice.

Moreover, to further accelerate the analysis, sources of uncertainty are evaluated only for those values that are used in the 2D matrix. This means that if the final value is obtained as the mean of a certain number of data, the uncertainty is calculated on the mean a posteriori, rather than resulting from error propagation.

The adopted methodology is the following:

- Select a number of indicators that we believe may cause uncertainty: year of determination, level of approximation, number of mines, number of sources;
- Assign to each indicator a certain scale, with lower values referring to higher reliability;
- Establish a minimum and a maximum uncertainty range.

Byproduct Fraction

Ideally the same methodology is applied to both byproduct fraction and value ratio, but it is chosen not to use it for byproduct fraction. The reason lies in the "level of approximation" indicator: assigning a weight to a sentence such as "almost all" seems as subjective as simply assigning an uncertainty interval to the whole byproduct fraction data. It is therefore decided to allocate uncertainty based on an overall overview of the different indicators. A four-levels scale is used ranging from very Low (± 0%), low (± 5%), medium (± 10%) to high (± 20%).

Value Ratio

For what concerns value ratio, instead, no single indicator is believed to be responsible alone for most of the uncertainty range. The first three indicators listed below are used for concentration ratio and the fourth one is related to efficiency ratio.

1. Temporal correlation: year of determination

2. Sample size: number of mines
3. Data source consistency: how many sources/calculation to obtain that data? Do different sources agree?
4. Reliability of efficiency ratio: since in many cases this data is not available, it is decided to evaluate case by case the reliability of the available value on a 3 levels scale. When efficiency ratio is not found, the highest level of uncertainty is assigned.

The obtained uncertainty matrix is shown in Table 4.

Table 4: Uncertainty matrix

Indicator \ Uncertainty	Very Low	Low	Medium	High
Temporal correlation (Year)	1 (After 2010)	1.5 (2000-2010)		2 (Before 2000)
Sample size (Number of sample)	1 (>20)	1.5 (5-20)	2 (1-5)	3 (1)
Data sources consistency (Number of data sources)	1 (1)		2 (>1)	
Efficiency ratio reliability	1	1.2		1.5

In order to convert the uncertainty matrix in a percentage value to assign as uncertainty, Equation 4.2.1 is used.

$$R = R_b \cdot \exp\left(\sqrt{[\ln\cdot(I_1)]^2 + [\ln\cdot(I_2)]^2 + [\ln\cdot(I_3)]^2 + [\ln\cdot(I_4)]^2}\right) \tag{4.2.1}$$

where $I_{1,2,3,4}$ are the values of different indicators and R_b is the basic uncertainty factor, which is chosen to be 5%. It follows that the range of uncertainty is:

Minimum: $R = 0.05 \cdot \exp\left(\sqrt{[\ln\cdot(1)]^2 + [\ln\cdot(1)]^2 + [\ln\cdot(1)]^2 + [\ln\cdot(1)]^2}\right) = 5\%$

Maximum: $R=0.05\cdot\exp\left(\sqrt{[\ln\cdot(2)]^2+[\ln\cdot(3)]^2+[\ln\cdot(2)]^2+[\ln\cdot(1.5)]^2}\right)=23\%$

With a range from 5 to 23%, the uncertainty analysis might seem conservative. In many cases, especially when the data refers to a specific geographical area and is used as world average, the chosen value is believed to potentially be incorrect of up to one order of magnitude. This reflects the difficulty of collecting data, in particular those related to companion metals. Therefore, the method developed here represents a relative measure of uncertainty with respect to each data point, rather than a strict statistical uncertainty like confidence intervals. Due to both a low concentration in the ores and a low economic contribute to overall companies' incomes, most of the time minor metals quantities are not reported in annual reports. With a fast growing demand mainly due to the electronic sector, more attention is expected to be given to minor metals and higher quality data to be published in the coming years.

The uncertainty obtained are used in the next chapter in order to test robustness of qualitative clustering.

4.3 Result of Categorizing

4.3.1 General Features

Once all the data have been collected, the 2D matrix is developed. Out of the 47 complete pairs available, 45 are inserted in the matrix, with byproduct fraction on the X-axis and value ratio on the Y-axis. It is decided not to report rhenium as a byproduct of copper-molybdenum ores and cobalt as byproduct of arsenic, due to their high value ratio. In both cases, in fact, the value is greater than 10, suggesting that referring to rhenium and arsenic as byproducts is inappropriate from an economic point of view.

The 2D matrix is reported in Figure 7, with the first element of each pair being the carrier.

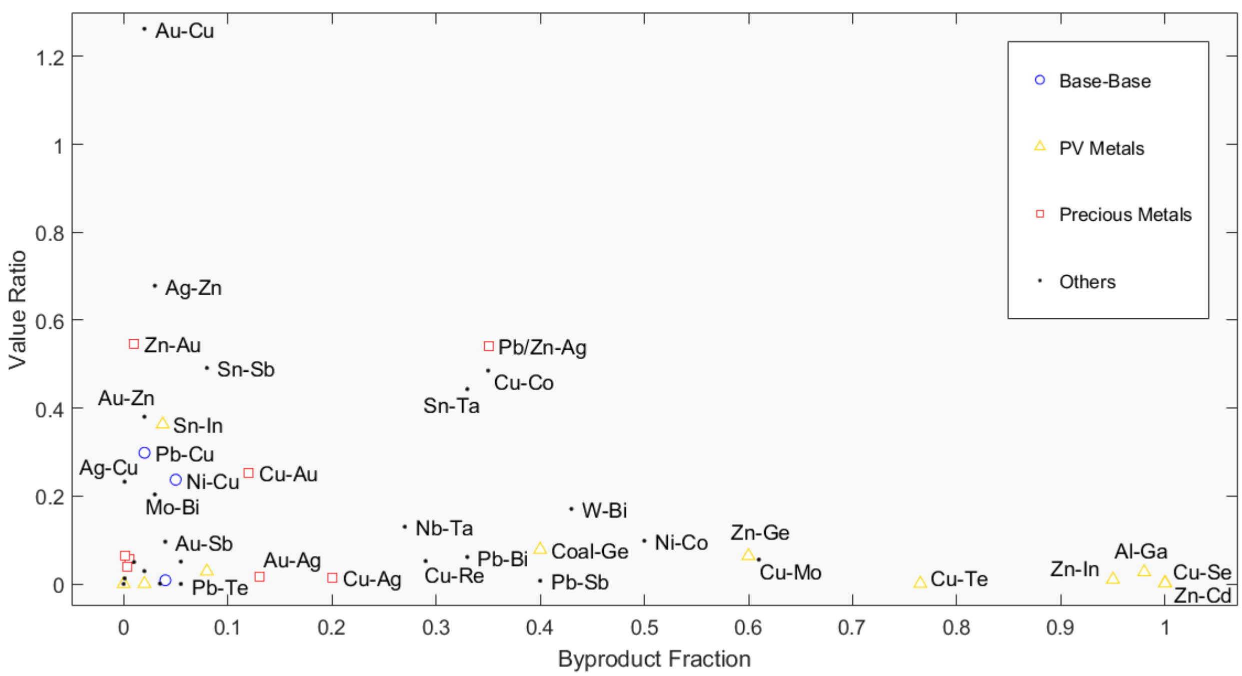


Figure 7: Categorization of 45 carrier-byproduct pairs

Pairs in the lower left corner all have similarly low byproduct fraction and value ratio and result indistinguishable. A zoomed version, from 0 to 0.1 in both axes, is provided in Figure 8.

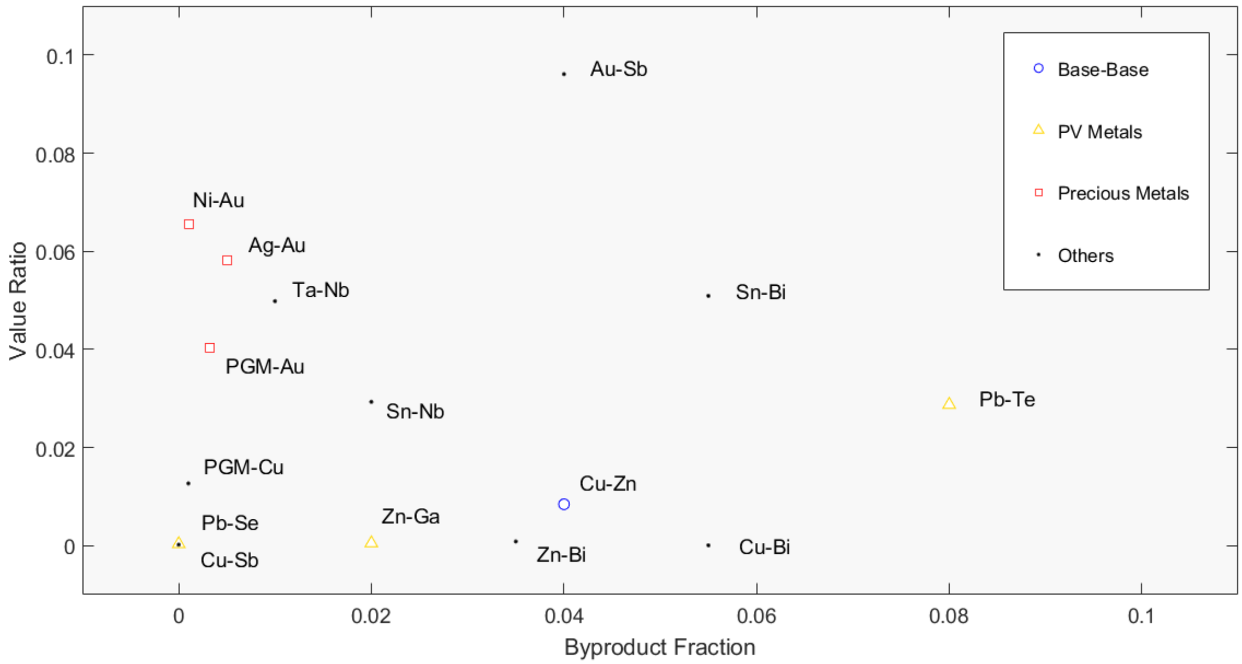


Figure 8: Categorization of 45 carrier-byproduct pairs - zoom in of the lower left corner

In two cases, multiple pairs share very close points in the matrix, therefore resulting indistinguishable one from each other. One case is in the lower left corner, where both Cu-Sb and Pb-Se have 0% byproduct fraction and very low value ratio, 0.0002 and 0.0003 respectively. One could argue that such points should not be included in the matrix, due to the fact that no production is today obtained from that specific carrier. However, as often happens, byproduct Fraction value changes over time and it might be that in some years, due to either economic or technological reasons, production of antimony from copper or selenium from lead rise again. It is therefore decided to include them in the matrix.

The second case is found in the lower right corner, where Zn-Cd and Cu-Se have both 100% byproduct fraction and a very low value ratio, 0.03 and 0.02 respectively.

The two axes represent concerns for different individual or companies. The Y-axe is related to metals producers, i.e. mining and refining companies, while the X-axe is connected to byproduct metals' consumers. Byproduct metals' consumers should not be confused with consumers of final

products. With the word consumer here we refer to producers and manufactures who use by-product metals in their production chain as row materials, such as PV cells manufacturers. Greater X values, i.e. a high byproduct fraction, may cause serious concerns among consumers of that particular byproduct. A shortage of the main carrier supply might, in fact, cause a shortage of the byproduct metal supply as well. It is therefore preferable for producers to either not use metals produced as byproduct or be sure that a substitute exists. Another risk for producers is the implementation of new extraction technologies for the carrier metals which may lead to lower or zero recovery of the byproduct. It is the case of tellurium obtained as byproduct form copper ores. Among the two method used to recover copper, only one allows the recovery of Te. In recent year, however, due to the higher achievable recovery of Cu using the other method, more and more operations are switching to this newer processing technique and tellurium is wasted.
On the other hand, a high value ratio is of concern for metal suppliers. If the value of the minor metal is of the same order of the carrier, miners would be affected by changes in byproduct demand and price. It is therefore essential for producers to understand the economic importance of byproduct metals in order to assess whether it's worth it or not to invest resources in their recovery.

In order to simplify the identification of the different metals, each pair is included in one of four groups, based on the nature of the Byproduct. The four groups are: Base-Base (both carrier and byproduct are base metals, i.e. Al, Co, Ni, Pb, Zn), PV Metals (Byproduct is used in PV cells manufacturing, i.e. Cd, Ga, Ge, In, Te, Se), Precious Metals (Byproduct is a precious metal, i.e. Ag, Au) and Others.
The choice of this groups is not random: it's expected that pairs belonging to the same group end up in similar location in the matrix. Base metal, for example, are expected to be obtained mainly as host metals and this is verified by the blue circles: all the three cases where a base metal is mined as byproduct of another base metal show a byproduct fraction smaller than five percent. On the other hand, minor metals used in PV cells are often referred in literature as critical due to their byproduct nature. As it can be seen from the matrix, the yellow triangles always appear in the lower part of the matrix, with Indium obtained from Tin as one single exception. This shows

that by having a small value ratio it is reasonable to consider them byproducts and this may cause criticality concerns. A clearer representation of such byproduct nature grouping is provided in the elementwise paragraph which follows.

4.3.2 Elementwise Results

Here results are presented per element, rather than per pair. If on one side, reporting data per pairs helps metal supplier to assess the importance of the minerals they mine, on the other it does not provide an instantaneous complete picture of the metal criticality. Indium for example is obtained 95% as byproduct from zinc and the remaining as byproduct of Tin. This is however not immediate in Figure 7, since the two point are far away from each other and the reader may take a while to realize that multiple points refer to indium as byproduct. By reporting the total fraction of primary metal obtained as byproduct, instead, this misunderstanding is avoided and the potential criticality of each element is easier to visualize. Figure 9 shows total byproduct fraction in relation to byproduct Herfindahl-Hirschman index (HHI).

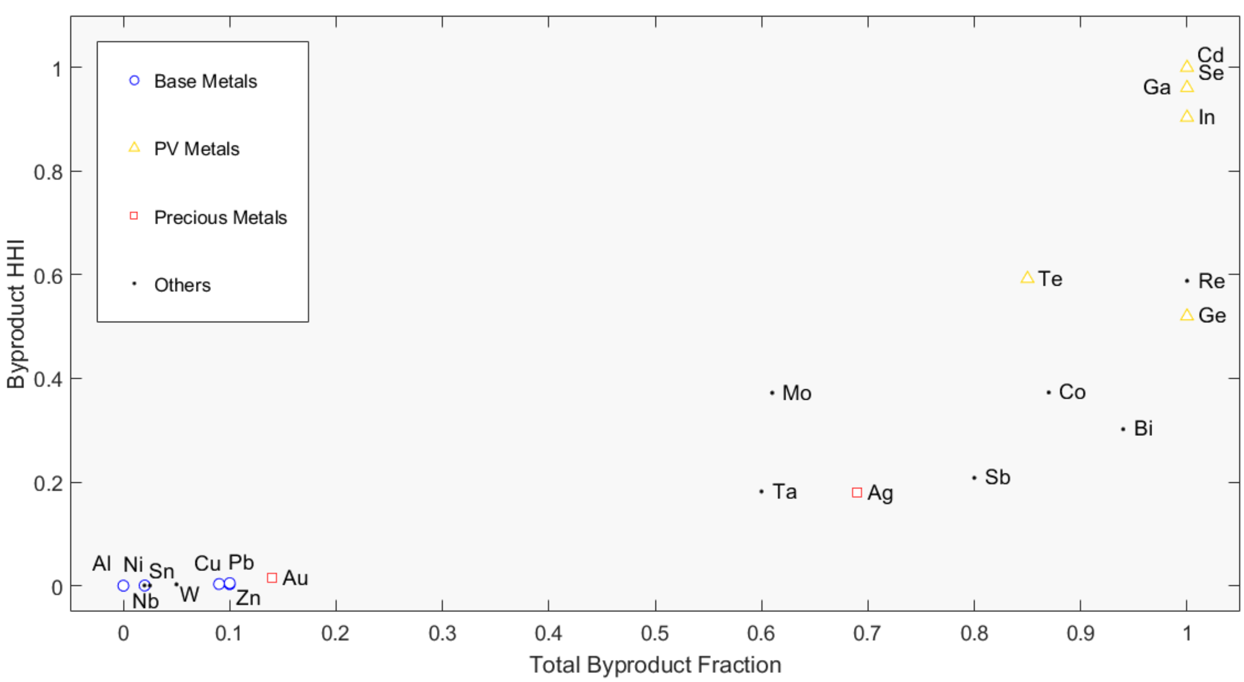


Figure 9: Elementwise categorization - Byproduct HHI over total byproduct fraction

By looking at the X-axis, it's clearer how base metals are mainly mined as main products, while PV related elements are usually mined entirely, or almost, as byproducts.

The Y axe represents the byproduct Herfindahl-Hirschman index. HHI was first used to understand the size of firms in a certain industry and is calculated according to Equation 4.3.2.1.

$$HHI = \sum_{i=1}^{n} F_i^2 \tag{4.3.2.1}$$

where n is the number of firms and F_i is the market share of the i-th firm in the industry
Therefore, high HHI means that a small number of firms are dominating the market, while a low HHI reflects a market with many competitors.

Similarly, byproduct HHI gives an idea how strong is the byproduct nature of the metal. It is calculated summing the squares of the byproduct fraction values from different carriers, as seen in Equation 4.3.2.2.

$$Byproduct\ HHI = \sum_{i=1}^{n} BP_i^2 \tag{4.3.2.2}$$

where n is the number of carriers and BP_i is the byproduct fraction from the i-th carrier.
A high byproduct HHI reflects the metals being mined mainly form one or few carriers and is related to greater concerns for consumers. Lower values, instead, reflect a byproduct which is either obtained mainly as primary metal or from a great number of carrier in similar quantities.

Four elements in the upper right corner, specifically cadmium, selenium, gallium and indium, have a byproduct HHI of 0.9 or higher and may result highly critical for consumers.
In order to provide an equivalent elementwise criticality matrix for the supply side, byproduct HHI is plotted over demand HHI and reported in Figure 10. Demand data are taken from the United States Geological Survey reports. When only US data are available, the reported demand sectors-shares are assumed to be valid also at global level. Demand HHI is calculated according to Equation 4.3.2.2.

$$Demand\ HHI = \sum_{i=1}^{n} D_i^2 \qquad (4.3.2.3)$$

where n is the number of sectors and D_i is the overall demand fraction of each i-th sector.

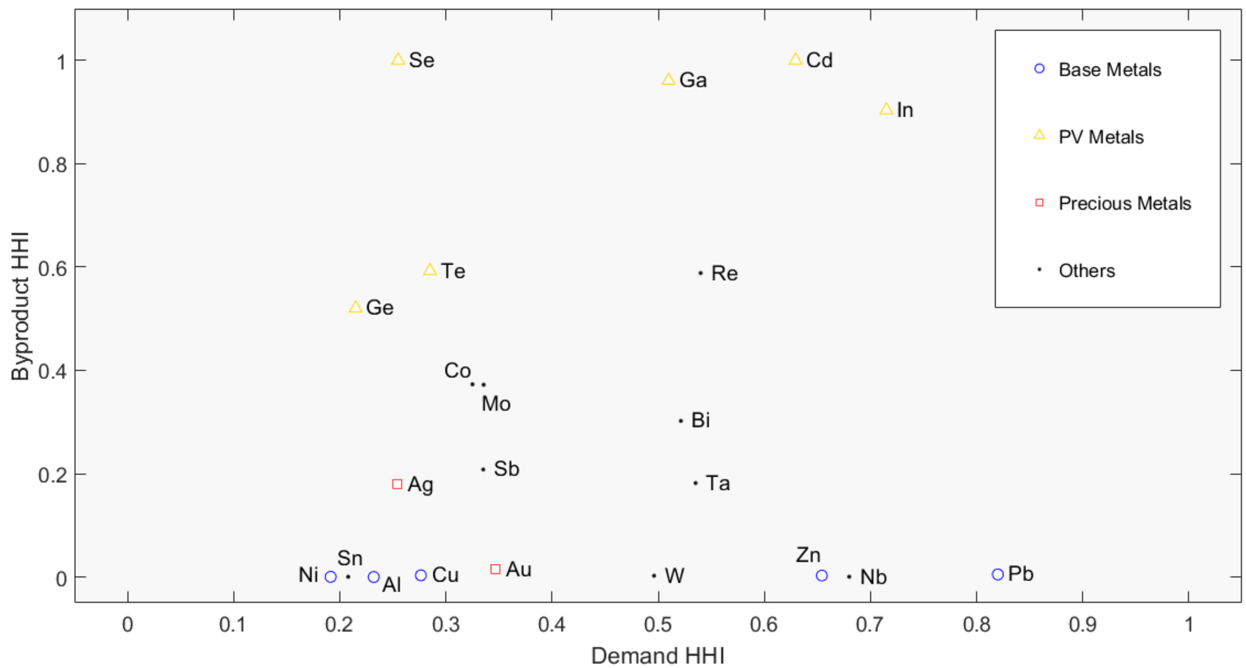


Figure 10: Elementwise categorization - Byproduct HHI over demand HHI

Element in the right part of the plot may cause major concern among producers. The high demand HHI, in fact, reflect the fact that the interested metal is used only in one or few sectors and therefore a technological switch may result in a rapid decrease in the demand.

4.3.3 Qualitative Grouping

A first, qualitatively grouping is presented in this paragraph. The aim is to group together different pairs which are believed to have similar criticality issues and which are expected to behave similarly from a market point of view.
Different pairs are clustered according to the category (producers or consumers) which might be more affected by criticality issues and the level of criticality assigned.

Five main groups are identified, as show in Figure 11. Two are critical for consumers, one for suppliers, one for both and one for none. The two groups referring to consumers differs due to different byproduct fraction: one in termed as medium criticality, the other as high.

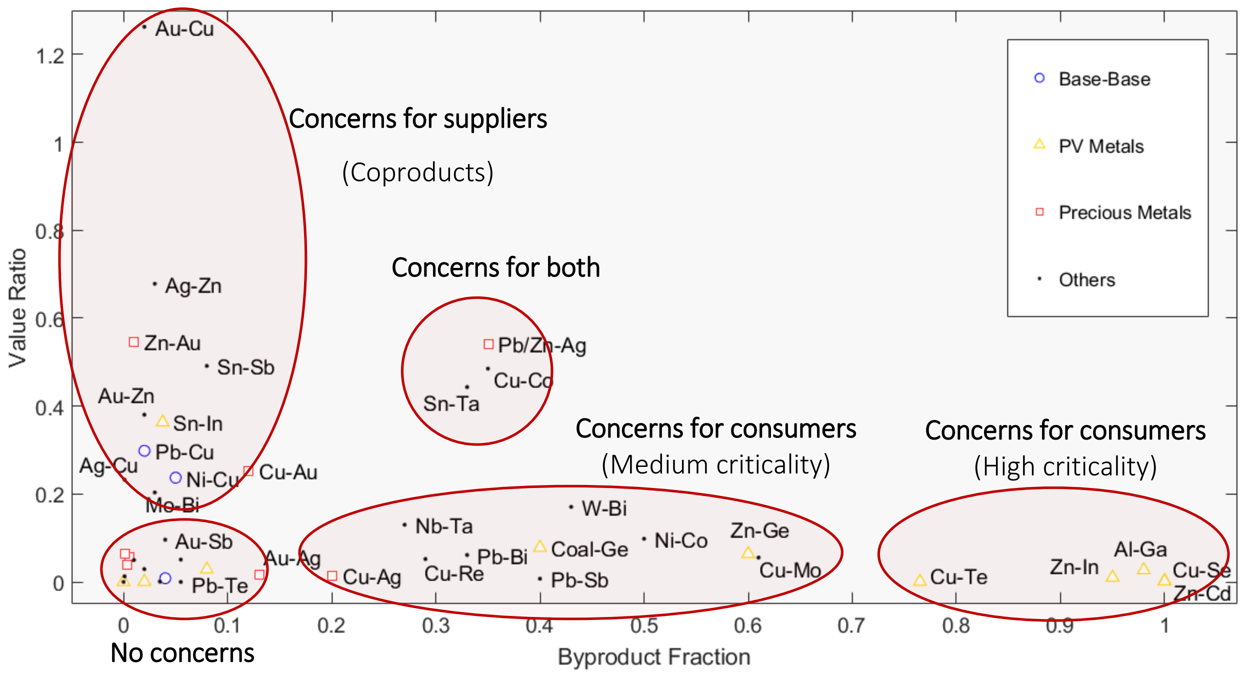


Figure 11: Categorization of carrier-byproduct pairs - Qualitative grouping

The points lying in the lower-left corner are labeled as "Not interesting". By having both a small byproduct fraction and value ratio, they do not significantly affect either producers or consumers. They are therefore excluded from the considerations that follows.

Pairs located in the lower part of the matrix are considered critical only for the consumers. By having small value ratio, in fact, they are of minor interest for the producers: most of the suppliers' revenues are related to carrier metals and therefore a decrease in demand of byproducts will not affect them economically. From the consumers' point of view, instead, high byproduct fractions do represent a source of major concern: a sudden rise of byproduct price would increase the cost of producing products that contain these byproduct metals, which would likely cause a decrease in sales. Two different clusters are identified and assigned different criticality: the first, covering byproduct range from 0.25 up to around 0.6 is assigned medium criticality, while the second, from 0.7 upwards, is labeled as highly critical. A decrease of production from one specific carrier, in fact, has different consequences depending where the pair is located. For pairs in the right side of the matrix, a decrease of byproduct supply would affect the vast majority of the market of such element, while this is not the case for pairs located in the medium criticality cluster. Let's pick molybdenum from copper as an example. Around 60% of world primarily molybdenum production is obtained from copper, with most of the remaining being mined independently of other carriers. If for some reason molybdenum recovery from copper drops while molybdenum demand remains unchanged, it's likely that suppliers who are producing molybdenum as primary metal will increase their mines' capacity. In this way, supply and demand are in equilibrium and price is not affected too heavily.

Pairs lying in the left part of the matrix, instead, are considered of concerns for suppliers. With value ratio of around 0.4 or greater, the minor metals make up for a large part of the total revenues and should therefore be considered as coproducts rather than byproducts. A sudden decrease in demand would negatively impact the producer's income causing the overall cost of extraction and refining to no longer been covered. From the consumers' point of view, instead, such pairs are not considered critical since most of the supply is obtained from different sources.

The final cluster is the one lying in the middle of the matrix, which includes Cu-Co, Sn-Ta and Pb/Zn-Ag. By having relatively high values both for byproduct fraction and value ratio they may be of concern for both suppliers and producers. However, due to the most likely coproduct nature, such pairs are considered to be more critical from the supply point of view, and only of medium criticality on the consumers' side.

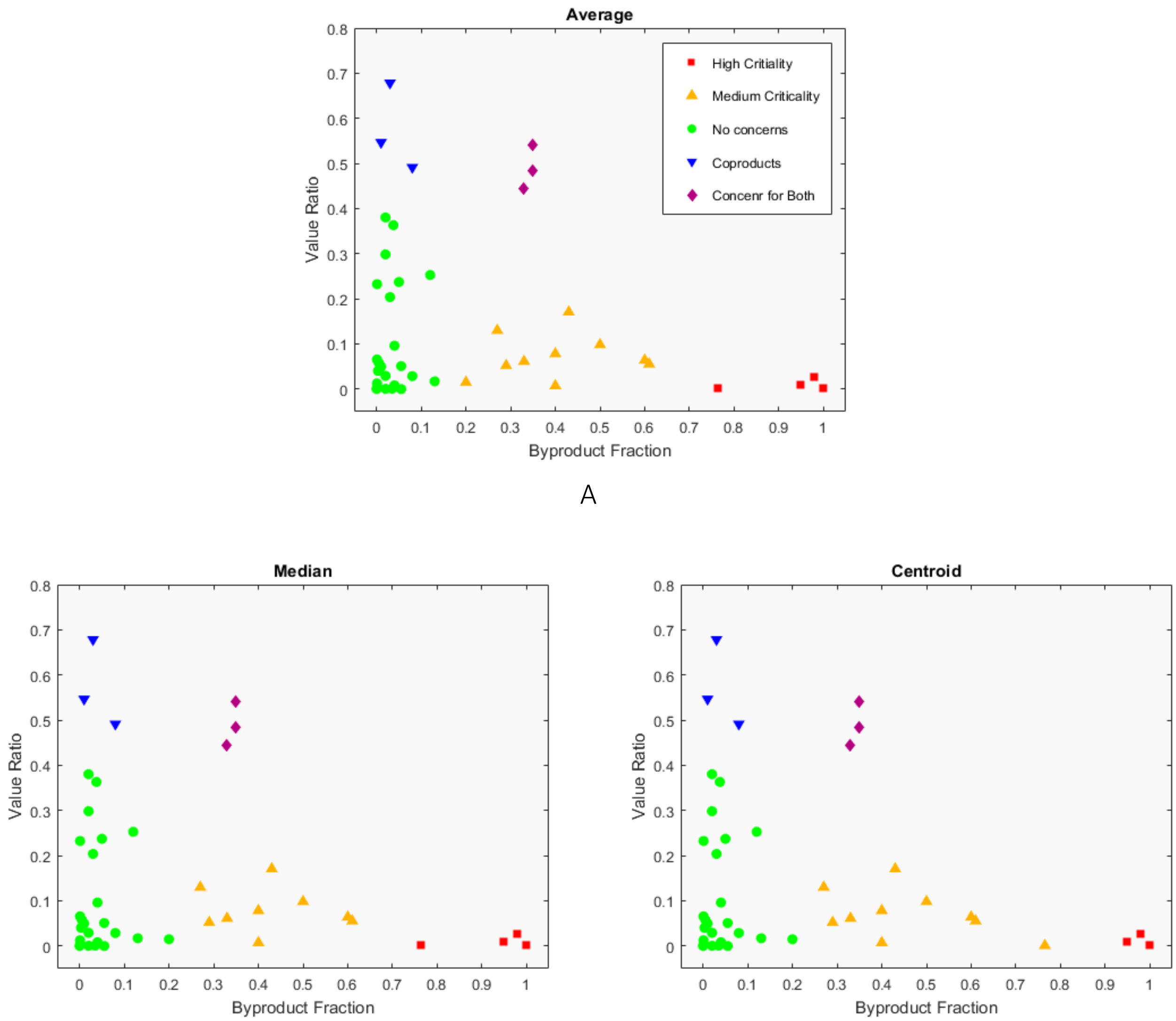
4.3.4 Quantitative Grouping and Results from Uncertainty Analysis

In this section, a quantitative clustering analysis is developed. Due to uncertainty in the data, the results should not be interpreted as totally accurate and definitive. Instead, they should be intended as a starting point for scholars interested in the field of statistical analysis.

First, hierarchical clustering is applied to the selected data, without taking uncertainty into considerations. Secondly, Monte Carlo simulation is integrated in order to test the robustness of the clustering in relation to data uncertainty. As explained in 'Uncertainty Analysis' section, the uncertainty ranges may have been underestimated and thus this second method should be considered with even more precautions.
The Au-Cu pair is chosen to not be included in this analysis because. Its location is far away from other data point in the 2D matrix and the clustering result would be heavily influenced by this single pair. With a value ratio greater than one (and byproduct fraction of 2%), its coproduct nature can safely be argued and the pair can be considered critical only on the supply side.

Agglomerative hierarchical cluster analysis is used. Each point starts as a separate entity and in the first step is merged to another point, forming a group or cluster. In the second step, each cluster emerged from the first step is merged to another one, forming a larger one. This procedure is repeated until the optimal number of groups is present. In order to decide which points or groups should be merged, the shortest distance is used. However, multiple definitions of distance are available and therefore multiple methods can be applied. These methods are referred as linkages. Some of the most commonly used are single linkage, complete linkage, average linkage, median linkage and centroid linkage. The first two can be considered as extremes, while the latter three lay somewhere in between.
Single linkage merge clusters based on the shortest distance between two pairs that are not yet belonging to the same cluster. The result in many cases are long thin clusters in which nearby elements have short distances, but pairs lying in opposite ends of the cluster may be further from each other than from elements of different groups. The exact opposite metric is complete linkage. In this case, the distance between groups is defined as the furthest distance between two points belonging to different groups. The result are more compact clusters with similar diameters.
The other three metrics are somewhere in between single and complete linkage, with distance between different cluster based on the average, the median or the centroid, as the names suggest.

This last three metrics are chosen and the maximum number of cluster fixed at five. The results are shown in Figure 12.



B C

Figure 12: Hierarchical clustering plots. A is based on average linkage, B on median linkage and C on centroid linkage

Two interesting observation can be drawn from the graphs above.

First, the results obtained with different metrics do not greatly differ greatly between each other. The same five clusters are always identified, with only two pairs, Cu-Ag and Cu-Te, not being consistent.

Secondly, the results are similar to those presented in the preliminary qualitative evaluation. If we consider average linkage, three out of five cluster are identical, with the only difference appearing in the left part of the matrix. Pairs having value ratio in the range 0.2-0.4 are in this case assigned to the same cluster as the “No concerns” pairs of the qualitative grouping. This highlights the non-triviality of a clear distinction between byproduct and coproduct. In this case, statistical methods seem to suggest that a value ratio up to 0.4 can be associated with a byproduct evaluation of the metal, while qualitatively a pair having a value ratio of 0.3 may already be considered a coproduction case.

The changes in either byproduct fraction or value ratio of each pair in relation to data uncertainty may affect the clustering results. Further analysis is therefore required in order to test robustness of the quantitative analysis. In order to assess how each single pair affects the clustering results, the effect of uncertainty is evaluated for one pair at a time.

By combining the byproduct fraction uncertainty range with the value ratio one, rectangular uncertainty ranges are obtained for each pair in the matrix, with the original value reported representing the center. In order to assess the impact of each pair, byproduct fraction and value ratio are changed so that the four corners of the rectangular uncertainty range are analyzed. A new clustering analysis, based on average linkage, is performed for each of the four different cases. If in at least one of the four cases the pair end up in a different cluster compare to the basic case, then a Monte Carlo simulation is performed. The simulation consists of the generation of 10 000 byproduct fraction-value ratio pairs within the uncertainty rectangle and the recording of the selected cluster in each repetition.

Monte Carlo simulation is required only for 7 out of 44 pairs; moreover, two statistically different clusters are identified only in two of this seven cases. Of particular relevance is tellurium as byproduct of copper which around 60% of the times belongs to the “High criticality” cluster, while in the remaining simulations ends up in the “Medium criticality” cluster. The other, less interesting case, is Cu-Re which switches from the “Not interesting” to the “Medium criticality” cluster, belonging to the first around 90% of the time.

Note that the results of Monte Carlo do not suggest the absolute confidence about which cluster a pair should belong to, but rather a relative measure of robustness with respect to uncertainty in our data. In this sense, with more than 95% of pairs always belonging to the same group, the clustering method looks robust. However, it has to be underlined once more how both the uncertainty and the clustering analysis should be considered as a starting point for scholars interested in the field and the results consequently carefully evaluated.

5 Metals Supply Elasticity and the Carrier-Byproduct Constraint

In order to assess the impact of byproduct mining as indicator of materials criticality, the different pairs reported in the 2D matrix need to be analyzed. In this section, pairs identified as "Highly Critical" are studied. In particular, indium as byproduct of zinc and tellurium as byproduct of copper are used as case studies. Price elasticity of supply is analyzed. As reported by Graedel et al., inelastic supply of byproducts may be caused by the carrier metals, which limit the supply of the byproduct. The aim of this chapter is thus to verify if this applies to indium and tellurium, given that the current researches do not understand this very well.

In order to estimate supply elasticity of indium and tellurium, econometric analyses are performed. In particular, three linear regression models are developed.

First, ordinary least squares model is used. The method relies on three basic assumptions: strict exogeneity, homoscedasticity of error terms and no autocorrelation of error terms. For what concerns homoscedasticity, White estimator is used to deal with this issue. However, if the other two assumptions are not satisfied, new models should be developed. In order to deal with autocorrelation, autoregressive distributed lag model is used and lag terms of dependent and independent variables are included. If strict exogeneity condition is not met, two-stages least squares technique is used in order to develop an instrument variable model. Furthermore, in order to test the validity of IV model compared to ARDL, three statistical tests are performed.

Detailed explanation of the different models and tests is provided in the methodology section, here equations are reported as a summary.

Ordinary Least Squares

$$Q_t^s = \alpha_1 P_t + \alpha_2' Z_t^s + \alpha_3' W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp P_t, Z_t^s, Z_t^d, W_t \tag{3.2.1.1}$$

$$Q_t^d = \beta_1 P_t + \beta_2' Z_t^d + \beta_3' W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp P_t, Z_t^d, Z_t^s, W_t \tag{3.2.1.2}$$

Autoregressive Distributed Lag

$$Q_t^s = \sum_{q=1}^{n_q} \alpha_q Q^s{}_{t-k} + \sum_{p=0}^{n_p} \alpha_{1,k} P_{t-p} + \sum_{z=0}^{n_z} \alpha'{}_{2,z} Z^s{}_{t-z} + \sum_{w=0}^{n_w} \alpha'{}_{3,w} W_{t-w} + \varepsilon_t^s, \qquad \varepsilon_t^s \perp P_t, Z_t^s, Z_t^d, W_t \tag{3.2.2.1}$$

$$Q_t^d = \sum_{q=1}^{n_q} \beta_q Q^d{}_{t-q} + \sum_{p=0}^{n_p} \beta_{1,p} P_{t-p} + \sum_{z=0}^{n_z} \beta'{}_{2,z} Z^d{}_{t-z} + \sum_{w=0}^{n_w} \beta'{}_{3,w} W_{t-w} + \varepsilon_t^d, \qquad \varepsilon_t^d \perp P_t, Z_t^d, Z_t^s, W_t \tag{3.2.2.2}$$

Instrumental Variable

$$Q_t^s = aP_t + a_2' Z_t^s + a_3' W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp Z_t^s, Z_t^d, W_t;\ corr(P_t, \varepsilon_t^d) \neq 0 \tag{3.2.3.1}$$

$$P_t = \gamma_1' Z_t^d + \gamma_2' W_t + \varepsilon_t \qquad \varepsilon_t \perp Z_t^d, Z_t^s, W_t; \tag{3.2.3.2}$$

$$\hat{P}_t = \gamma_1' Z_t^d + \gamma_2' W_t \tag{3.2.3.3}$$

$$Q_t^s = \alpha_1 \hat{P}_t + \alpha_2' Z_t^s + \alpha_3' W_t + \varepsilon_t^s \qquad \varepsilon_t^s \perp Z_t^s, Z_t^d, W_t \tag{3.2.3.4}$$

$$Q_t^d = bP_t + b' Z_t^d + b' W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp Z_t^d, Z_t^s, W_t;\ corr(P_t, \varepsilon_t^d) \neq 0 \tag{3.2.3.5}$$

$$P_t = \delta_1' Z_t^s + \delta_2' W_t + \varepsilon_t \qquad \varepsilon_t \perp Z_t^s, Z_t^d, W_t \tag{3.2.3.6}$$

$$\hat{P}_t = \delta_1' Z_t^s + \delta_2' W_t \tag{3.2.3.7}$$

$$Q_t^d = \beta_1 \hat{P}_t + \beta_2' Z_t^d + \beta_3' W_t + \varepsilon_t^d \qquad \varepsilon_t^d \perp Z_t^d, Z_t^s, W_t \tag{3.2.3.8}$$

5.1 Case Study on Indium

5.1.1 Data Description

Time series data are collected for each of the interested variables. The time span is 1972 to 2011, with both lower and upper limit due to indium specific data.

In quantity data is responsible for the lower limit. Indium world production data are based on USGS DS140 time series, which starts reporting in 1972 (US Geological Survey Tellurium Statistics 2016). Note that US production is not included in the original time series due to withheld information of the US sole primary producer. It is decided to add it manually through assumptions based on USGS Indium Mineral Commodity Summaries. Even though the exact production is withheld, in fact, the trend is still reported. It is therefore possible to estimate how the US primary production varies between 1972 and 1993, in which 1993 was the last year of operation of the only producer. Indium quantity time series is therefore obtained by adding US estimated production to the USGS DS140 time series.

On the other hand, it is found that the price trend in recent years was dominated not by the supply and demand relation. In particular, a scam started in 2012 in China, known as the Fanya scam, caused a peak in price followed by a rapid decrease. Therefore, it is decided to use 2011 as the most recent year in the analysis. Indium price data are obtained from the "Metal Prices in the United States Through 2010" report from USGS and represent the annual average US producer price for 99.97%-purity metal (U.S. Geological Survey 2013).

Time series of indium price and quantities are provided in Figure 13.

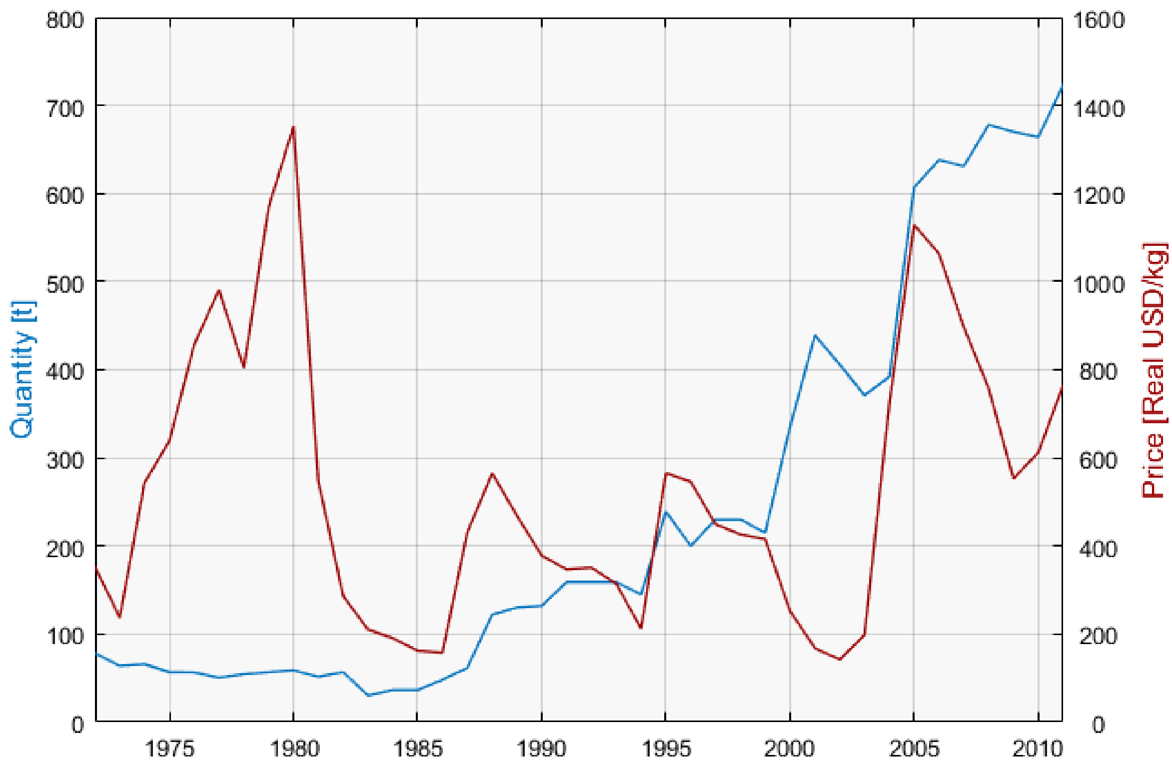


Figure 13: Indium quantity and price time series (1972-2011)

Besides indium price and quantity, the data of interest can be divided in three groups:

- Supply Shifters
- Demand Shifters
- Common Shifters

Supply shifters

As introduced in the 'Methodology' chapter, supply shifters are variables which affect indium supply in ways that are uncorrelated to its demand. The most trivial examples are zinc related variables. While indium supply is strictly linked to zinc production due to its byproduct nature, In demand is not affected by it. The vast majority of indium demand is linked to indium tin oxides (ITO), used in flat panel displays (FPD) and other electronic devices such as PV cells, and no use is reported in zinc alloys. Therefore, In demand can safety be considered independent from zinc data. Among all available zinc-related variables (i.e. supply, demand, price, etc.), supply is the only one considered relevant for indium. Zn supply time series, obtained from USGS Zinc DS140 is shown in Figure 14.A.

Another supply shifter that should be included in the model is something reflecting level of industrial production (IP). IP is an economic indicator which measures the production output of major sectors, such as manufacturing, mining and utilities and it is believed to reflect the changes of metals supply in general (Board of Governors of the Federal Reserve System 2011). As indium is produced in many different countries, a global index would be the best option. A world index is not available and therefore US IP (United States IP), G7 IP (G7 countries IP) and OECD IP (OECD, Organization for Economic Co-operation and Development IP) are chosen. In this way, however, around 50% of today's world indium production is not represented, as China is excluded from all the mentioned indexes and it accounts for around half of world supply. China GDP in Mining, Manufacturing, Utilities (CMMU), somehow similar to industrial production indexes, is added in order to deal with such issue. US IP index is obtained from the Board of Governors of the Federal Reserve System (known as Federal Reserve Board), G7 and OECD IP are taken from the OCED official website, while CMMU is available online at the United Nations National Accounts Database, in USD (United Nations 2015) . The three IP indexes are shown in Figure 14.B, while China MMU is presented in Figure 14.C.

Last, but not least, interest rate is included. According to Hotelling's rule, in fact, mineral resources can be regarded as capital asset (Hotelling 1931). Therefore, low interest rates reduce the cost of capital and increase supply. Multiple indexes are available from the Board of Governors of the Federal Reserve System; 1, 5 and 10 year interest rate, based on the annual average value, are chosen and their trend illustrated in Figure 14.D.

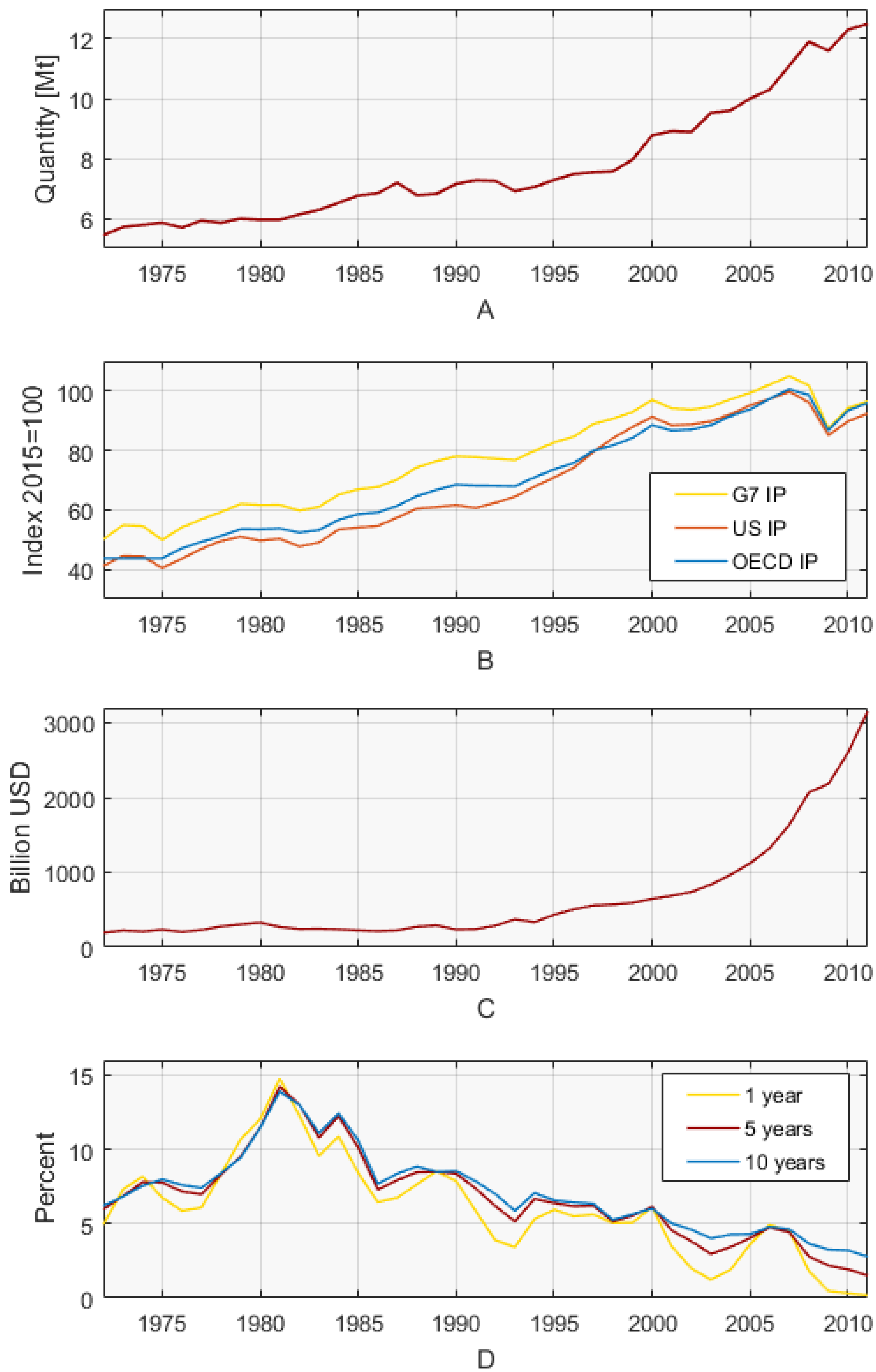


Figure 14: Indium supply shifters. A represents zinc supply, B the various IP indexes, C is China MMU index and D the three interest rates

Demand Shifters

Similarly to supply shifters, demand shifters are variables which are related to indium demand but do not affect the supply curve.

Ideally, one data series for each of the major demand sectors should be included in the model.

Indium has three main applications: as ITO in coatings, as metals alloy in solders and as semiconductor material in electrical components. Use in thin conductive oxides correspond to approximately 80% or more of overall demand and therefore a demand shifter related to it is necessary. Demand of indium in flat panel displays (FPD) production is used as a proxy for ITO demand. As FPD is the main application of ITO, demand in this application is believed to be a good demand shifter. The time-series related to In use in FPD is presented in Figure 15.A, adapted from a 2012 Nyrstar presentation (Constant 2012).

In addition to the specific demand-sector data discussed above, two more indicators are chosen as demand shifter. The first, connected to income level, is world GDP collected from the US Bureau of Economic Analysis. The second is a stock market index, known as Standard and Poor's 500 (S&P 500), which is considered an indicator of business cycles. It's developed by S&P Dow Jones Indices and available on Yahoo! Finance (Yahoo! Finance 2015). The data collected are annual, based on the average of monthly closing value.

Both World GDP and S&P500 time series are presented in Figure 15.B and 15.C respectively.

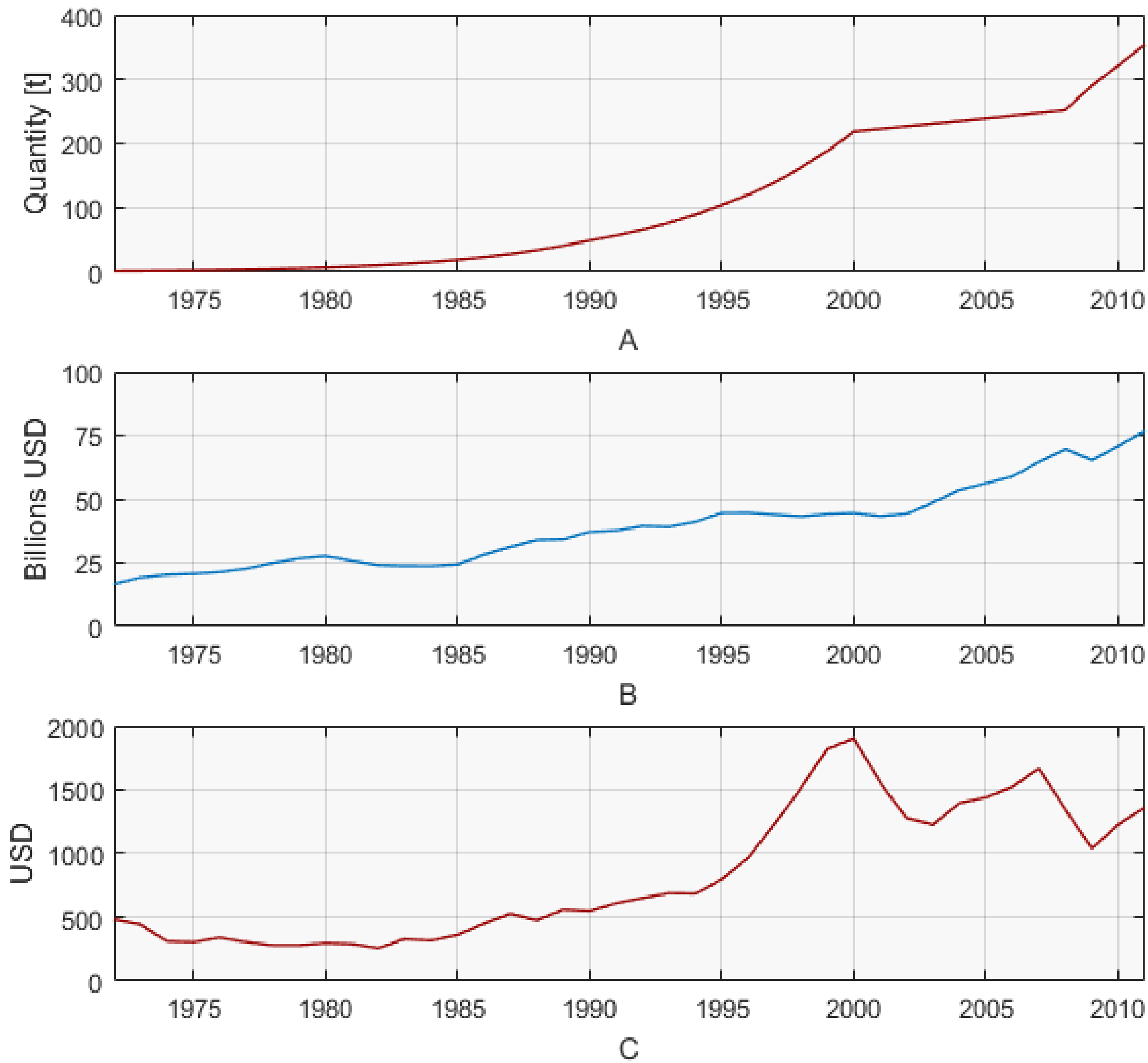


Figure 15: Indium demand shifters. A represents use of indium in flat panel display. B is world GDP. C represents S&P500 index.

Common Shifters

Common shifters are indicators which affect supply and demand simultaneously. The only included shifter is time. On the supply side this can be seen as an indicator that capture all implicit technological improvement, while on the demand it is used to represent unobserved demand shift such as population growth. A constant term is also included, which value is determined by the regression.

Whenever a value is given in monetary unit, it’s deflected using world real GDP deflator form US Bureau of Economic Analysis. Moreover, quantities and monetary data are log transformed, as suggested by many econometricians (Stock and Watson 2003).

5.1.2 Results & Conclusions

Ordinary Least Squares Model

The first step of the OLS model consists of the selection of the most appropriate shifters. When multiple variables describe the same supply or demand shifter, in fact, the number can be reduced by choosing the ones showing higher correlation with the dependent variable. It is the case for interest rate and IP index.

For what concerns interest rate, the 10y one is found to have the highest correlation with supply, -0.85. The negative value is as predicted by Hoelling’s rule: an increase of interest rate is followed by a decrease of supply.

The level of industrial production is found to positively affect supply. In particular, OECD (0.94) and China MMU (0.91) are moved forward. It is decided to keep both indexes in order to capture as much of the supply as possible. If CMMU was not included, in fact, around 50% of supply would be not be described.

Once the number of variables have been reduced, the supply and demand equations are as follow, where c is the constant term, previously included in the W term.

$$Q_t^s = c + \alpha_1 P_t + \alpha_{2a}(Q_{Zn})_t + \alpha_{2b}(OECD\ IP)_t + \alpha_{2c}(CMMU)_t + \alpha_{2d}(Int.10y)_t + \alpha_3 t + \varepsilon_t^s \tag{5.1.2.1}$$

$$Q_t^d = c + \beta_1 P_t + \beta_{2a}(GDP)_t + \beta_{2b}(S\&P500)_t + \beta_{2c}(FDP_{demand})_t + \beta_3 t + \varepsilon_t^s \tag{5.1.2.2}$$

OLS is performed and the results of the regression reported in Table 5 and Table 6.

Table 5: OLS model summary for indium supply

	Dependent variable:
	In.quantity
In.price	0.084
	(0.083)
Zn.quantity	−0.733
	(0.884)
OECD.IP	0.029**
	(0.013)
China.MMU	0.254
	(0.205)
Interest10y	−0.115***
	(0.024)
Year	0.015
	(0.025)
Constant	−21.532
	(43.962)
Observations	40
R^2	0.946
Adjusted R^2	0.936
Residual Std. Error	0.254 (df = 33)
F Statistic	95.524*** (df = 6; 33)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 6: OLS model summary for indium demand

	Dependent variable:
	In.quantity
In.price	−0.045
	(0.109)
World.GDP	1.995**
	(0.767)
SP500	0.887***
	(0.146)
FPD.demand	−0.511***
	(0.134)
Year	0.040
	(0.028)
Constant	−141.086***
	(41.188)
Observations	40
R^2	0.947
Adjusted R^2	0.939
Residual Std. Error	0.248 (df = 34)
F Statistic	120.513*** (df = 5; 34)
Note:	*p<0.1; **p<0.05; ***p<0.01

White estimator is used to correct for small sample proprieties. The estimation and inference of demand and supply elasticity are as follow:

- Supply: 0.08, 95% confidence interval (CI) [-0.10,0.27]
- Demand: -0.04, 95% CI [-0.25, -0.16]

Given the small values of the coefficient of price, it is argued that supply and demand are both inelastic to price. As introduced in the methodology section, steel is known to be price inelastic to supply and the coefficient found in that case is larger than those found for indium (-0.2 to -0.3).

Moreover, the price coefficient ranges of supply and demand cover zero and this is considered a stronger evidence of the argued inelasticity.

Autoregressive Distributed Lag Model

In cases where the error terms exhibit autocorrelation, lag terms of variables should be included. The OLS model is used a starting point and the number of variables is reduced using forward, backward and both direction stepwise selection. The results are the same in the three cases: China MMU and Zn quantity are removed from the supply equation, while the demand equation is left unchanged.

The residuals of the stepwise regression are analyzed to understand if there is autocorrelation.

As shown in Figure 16 and Figure 17, autocorrelation is present in the supply model, while no strong evidences are found for the demand model. It is therefore necessary to add lags terms to the supply equation.

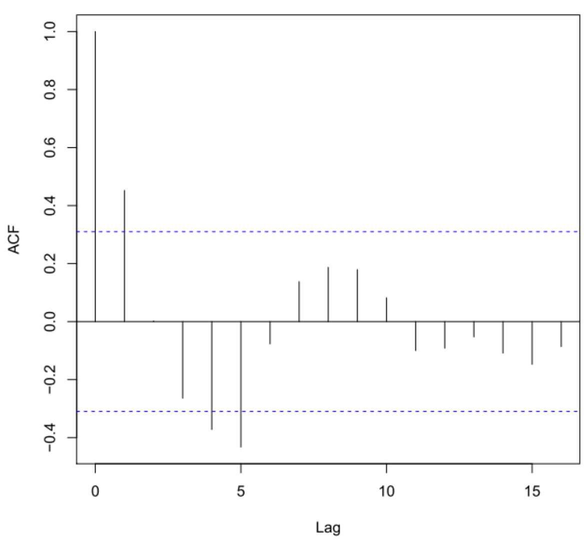


Figure 16: Autocorrelation of residuals of stepwise regression of indium supply

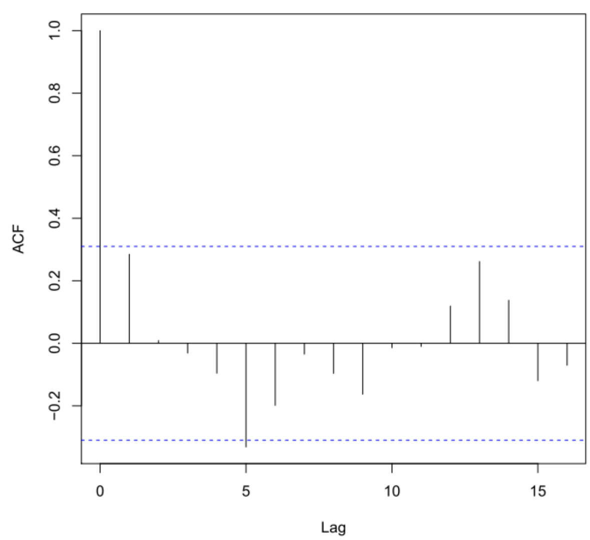


Figure 17: Autocorrelation of residuals of stepwise regression of indium demand

In order to understand which orders of lags to include, VAR model lag order selection is used. The results suggest that only the first order lag terms should be included. As explained in the methodology section, VAR method does not suggest which variable's lags term should be added. All the m^{n+1} possible combinations are analyzed and the one showing the highest adjusted R^2 is chosen (where n is the number of variables, excluding year and the constant term). The one including 1st lag of indium quantity, interest rate and OECP IP index, is selected. The summary of the regression of the selected model is shown in Table 7.

Table 7: ARDL model summary for indium supply

	Dependent variable:
	In.quantity
Year	0.004
	(0.017)
In.price	0.056
	(0.056)
OECD IP	0.006
	(0.012)
Interest 10y	−0.004
	(0.034)
Inq1	0.464***
	(0.118)
OECD1	0.016
	(0.013)
Int1	−0.053
	(0.037)
Constant	−5.741
	(33.039)
Observations	39
R^2	0.973
Adjusted R^2	0.968
Residual Std. Error	0.182 (df = 31)
F Statistic	162.646*** (df = 7; 31)
Note:	*p<0.1; **p<0.05; ***p<0.01

Residuals are analyzed again, in order to verify whether the autocorrelation problem has been solved. As seen in Figure 18, autocorrelation is no longer present.

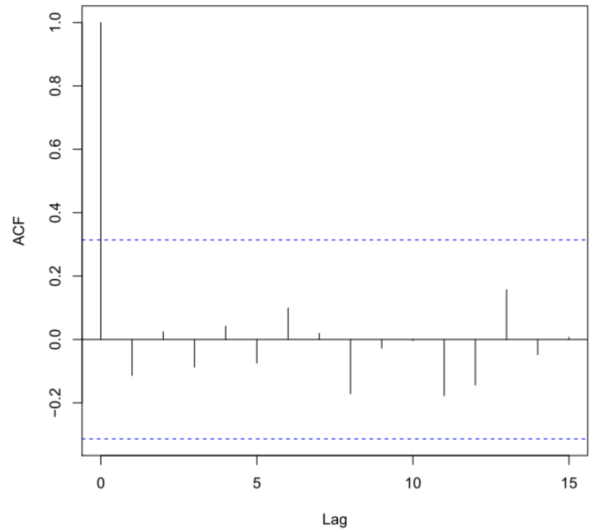


Figure 18: Autocorrelation of residuals of ARDL model for indium supply

Due to the presence of lag terms, long run elasticity is evaluated, as explained in the methodology section. Long run elasticity is calculated according to Equation 3.2.2.3, using the White estimator to deal with small sample proprieties.
Estimation and inference of long run elasticity for supply is as follow:

- 0.11, 95% CI [-0.08, 0.29]

Also in this case price coefficient is small and the confidence interval covers zero. It is therefore argued that supply is inelastic.

Instrumental Variable Model

Instrumental variable model is developed in order to deal with endogeneity of price. First, price is regressed over a set of instruments, which for the supply equation are the demands shifters and vice versa. Then the obtained estimator of price is inserted in the previous model and the regression performed. The starting point of the second stage for the supply equation is the ARDL model, while for demand OLS model is used, since no serial correlation is observed in the OLS model. The results of the four regressions are shown here, with Table 8 and 9 referring to the supply, and Table 10 and 11 to the demand.

Table 8: First stage of IV model summary for indium supply

	Dependent variable:
	In.price
Year	−0.090**
	(0.042)
World.GDP	5.199***
	(0.817)
SP500	0.298
	(0.246)
FPD.demand	−0.698***
	(0.187)
Constant	23.400
	(65.530)
Observations	39
R^2	0.657
Adjusted R^2	0.617
Residual Std. Error	0.389 (df = 34)
F Statistic	16.296*** (df = 4; 34)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 9: Second stage of IV model summary for indium supply

	Dependent variable:
	In.quantity
In.price	0.076
	(0.071)
Year	0.004
	(0.017)
OECD.IP	0.006
	(0.012)
Interest10y	−0.008
	(0.036)
Inq1	0.459***
	(0.119)
OECD1	0.016
	(0.013)
Int1	−0.048
	(0.038)
Constant	−7.756
	(33.412)
Observations	39
R^2	0.973
Adjusted R^2	0.967
Residual Std. Error	0.182 (df = 31)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 10: First stage of IV model summary for indium demand

	Dependent variable:
	In.price
Year	−0.051
	(0.053)
OECD.IP	0.026
	(0.039)
Interest10y	0.202*
	(0.103)
Inq1	0.271
	(0.372)
OECD1	−0.008
	(0.041)
Int1	−0.248**
	(0.108)
Constant	104.611
	(103.289)
Observations	39
R^2	0.288
Adjusted R^2	0.155
Residual Std. Error	0.578 (df = 32)
F Statistic	2.160* (df = 6; 32)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 11: Second stage of IV model summary for indium demand

	Dependent variable:
	In.quantity
In.price	−0.256
	(0.245)
Year	0.014
	(0.036)
World.GDP	3.241**
	(1.383)
SP500	0.859***
	(0.179)
FPD.demand	−0.599***
	(0.211)
Constant	−125.551***
	(44.025)
Observations	39
R^2	0.943
Adjusted R^2	0.934
Residual Std. Error	0.259 (df = 33)
Note:	*p<0.1; **p<0.05; ***p<0.01

Estimation and inference of long run elasticity are as follows:

- Supply: 0.14, 95% CI [-0.11, 0.39]
- Demand: -0.26, 95% CI [-0.70, 0.19]

Three tests are performed in order to validate this model compared to the ARDL one. The results are shown in Table 12.

Table 12: IV model tests summary for indium

	Supply	Implication	Demand	Implication
F Test	p<0.01	Instruments are correlated with price	p<0.01	Instruments are correlated with price
Sargan–Hansen test for exogeneity	p=0.247	No statistical evidence of instruments being endogenous	p=0.77	No statistical evidence of instruments being endogenous
Wu-Hausman test for endogeneity	p=0.646	ARDL as consistent as IV	p<0.01	ARDL is statistically not as consistent as IV

Concerning F test, the small p value in both cases suggest a good correlation between the instruments and price. The first condition for good instrument is therefore verified.

The Sargan-Hansen test provides in both cases a p value greater than the significance level (0.05) and therefore the null hypothesis cannot be rejected. There is no statistical evidence that suggest endogeneity among the chosen instruments. Goodness of instruments is therefore verified.

The Wu-Hausman test provides different results in the supply and demand cases. For what concerns supply, the large p value suggests that the null hypothesis cannot be rejected. This implies that there is no statistical evidence that IV model provides a better estimation compared to ARDL model. Moreover, given that goodness of instrument has been verified, endogeneity of price is considered responsible for this. For what concerns demand, instead, the null hypothesis can be rejected and the IV model used to calculate price elasticity of demand.

Summary of three methods

In Table 13, a summary of the results of the three methods is presented. From the analysis ARDL is found to be the most consistent among the three methods in order to determine price elasticity of supply. Price is argued to be inelastic, given the small estimated value

Table 13: Summary of supply elasticity results for indium

	α_1	Standard Error	95% Confidence Interval
OLS	0.08	0.10	(-0.10, 0.27)
ARDL	0.11	-	-
IV	0.14	0.13	(-0.11, 0.39)

Discussion

In order to verify whether the carrier metal-zinc-is responsible for the inelastic behavior of supply, potential supply of indium is plotted together with indium supply in Figure 19. The potential supply time series is obtained considering 100% recovery of all indium available in zinc ores. For this purpose, a concentration ratio of 0.00004, as presented in appendix x, is used.

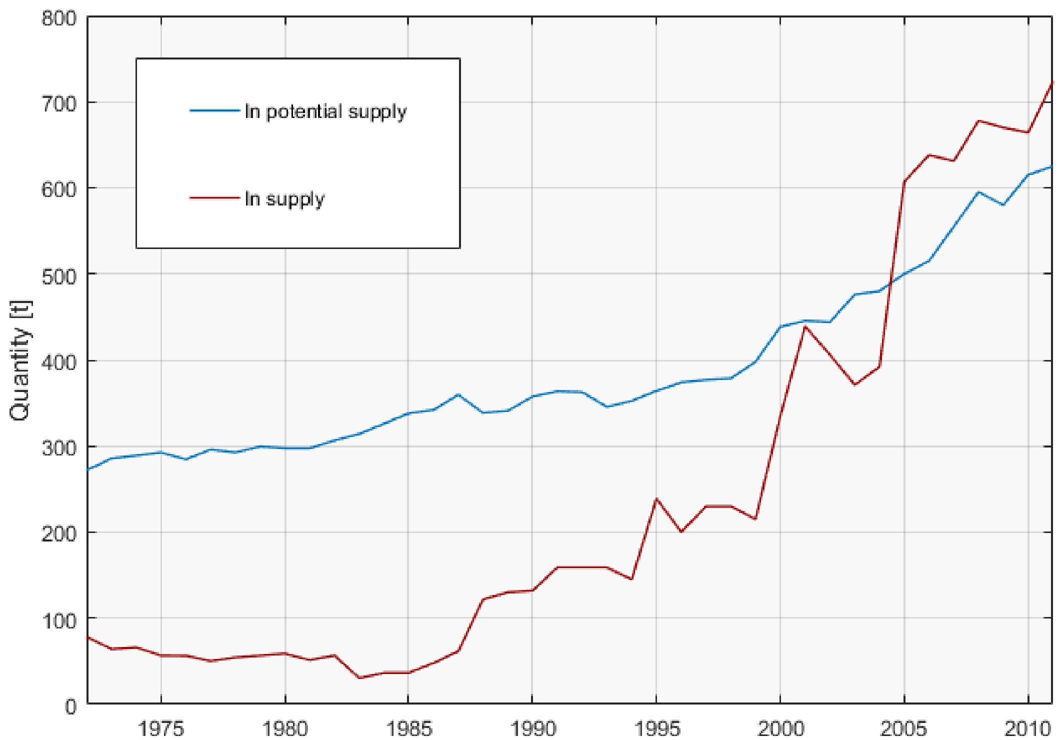


Figure 19: Indium supply and maximum potential supply from zinc ores (1972-2011)

As shown in Figure 19, starting from the early ‘80s, indium supply has grown significantly while the maximum potential supply from the main carrier-zinc-has grown at a slower rate. Indium supply is therefore approaching maximum supply potential and it can be argued that the limitation set by the carrier is the cause of supply of indium being price inelastic.

The fact that in recent year indium supply have surpassed its potential supply from zinc ores is explained by the increased recycling of the byproduct metal. Indium supply data, as said, is taken from USGS DS140 which reports indium production as “refinery production” and for some countries this includes secondary production. Moreover, the increase of secondary production can be seen as an evidence of lacking of supply from zinc ores. In fact, according to a report from a consulting company, 60 to 65% world indium supply comes from secondary sources in 2012 (Vulcan 2013).

The discrepancy between potential supply from zinc and actual supply of indium is therefore justified. In addition, indium is produced also from tin and as seen by the byproduct fraction in Figure 7, around 4% of primary production is due to this carrier.

5.2 Case Study on Tellurium

5.2.1 Data Description

Similar approach as the one used for indium is adopted for tellurium. First, for each variable of interest, time series are collected. In this case, the time span is 1960-2003, again limited by quantity and price data.

Contrary to the indium case, price affects the starting year of the analysis. Te price data are considered not reliable before the 60's, since only two different values are shown between 1931 and 1954. The source is USGS DS140 tellurium time series (US Geological Survey Tellurium Statistics 2016).

On the other end, Te quantity fix the most recent year to 2003. The estimation of tellurium world total primarily supply has never been an easy task, but in recent years the estimations made by two of the most widely trusted organizations for this type of data are considered to be unreliable. Both the United States Geological Survey and the British Geological Survey, in fact, do not report supply from main producers such as China, Germany, Belgium and others, with the result of a heavily underestimated world total.
BGS data are chosen for the analysis, given the fact that a greater number of producing countries, included the US, are reported in comparison to USGS DS140 time series (British Geological Survey 2016).

Time series of both tellurium quantity and price are presented in Figure 20.

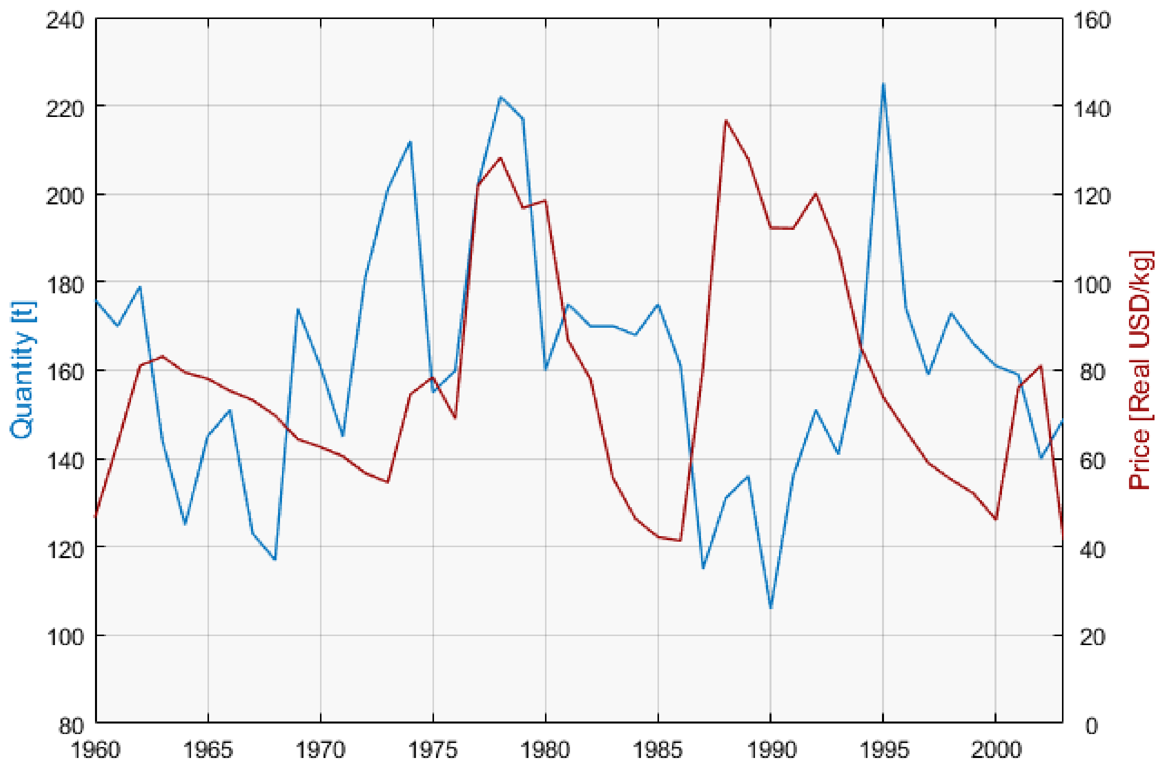


Figure 20: Tellurium quantity and price time series (1960-2003)

Supply Shifter

In the case of tellurium, copper related variables are chosen as supply shifters. The vast majority of tellurium demand is linked to steel, electronics and rubber, and only a very small percentage is used in copper alloys. Te demand can safety be considered independent from copper and therefore Cu data used as supply shifters. Once again, supply is the only copper-related variable considered. Moreover, tellurium is mainly recovered from the anode slimes originated during pyro metallurgical production of copper and therefore only this quantity, also referred as copper concentrates, is considered (Kavlak and Graedel 2013b). Copper concentrates supply time series, obtained from the International Copper Study Group (ICGS) 2015 Copper Factbook, is shown in Figure 21.A (International Copper Study Group 2015).

Industrial productions indexes and interested rate are also included and their trend as shown in Figure 21.b and 21.C respectively. In this case, US IP and G7 IP are the only two required, since most of the production is done by member of the G7. Concerning interest rates, 1, 5 and 10 years interest rate from the Federal Reserve Board are used.

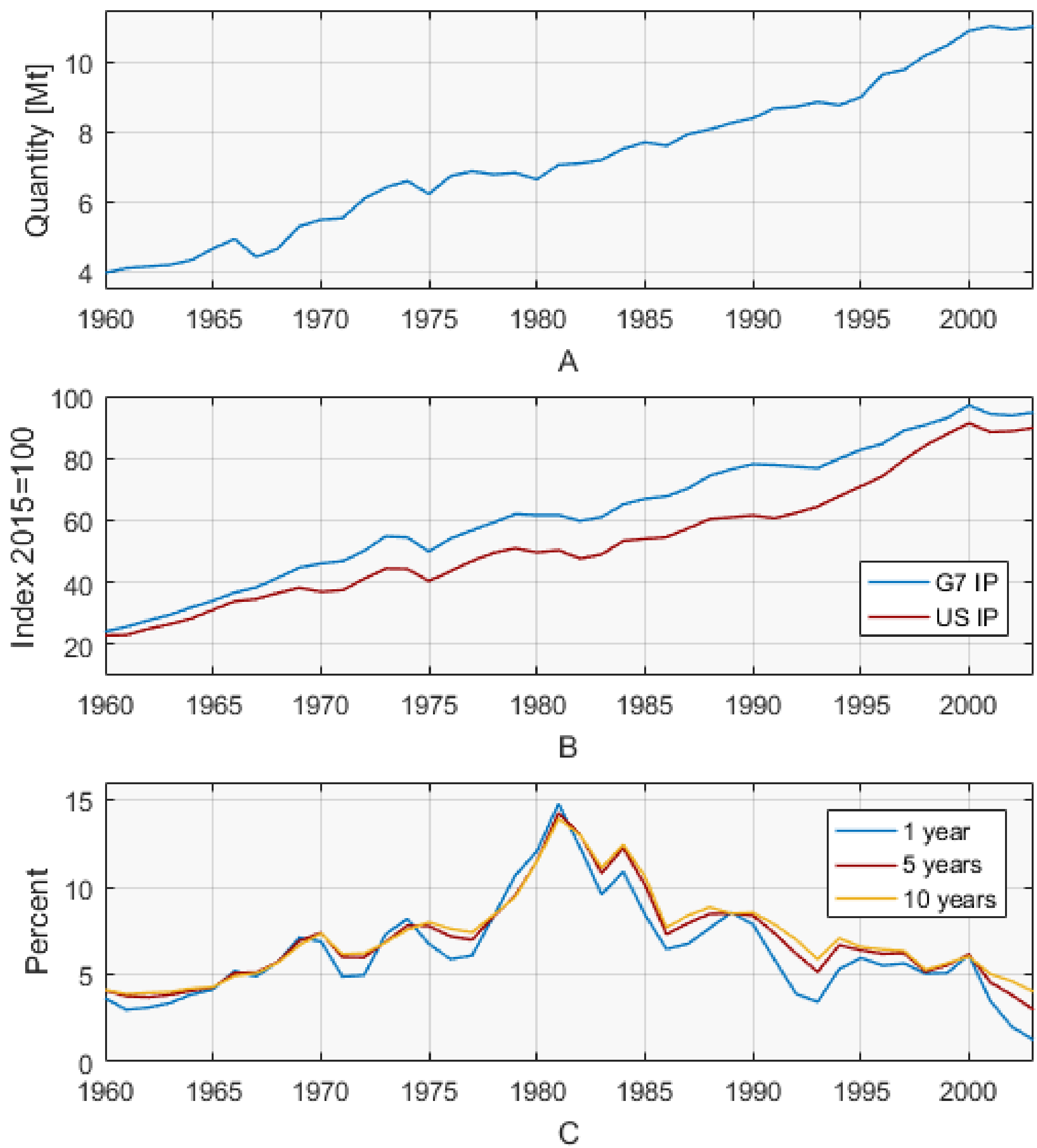


Figure 21: Tellurium supply shifters. A represents copper concentrates supply, B the two IP indexes used and C the three interest rates

Demand Shifters

Again, one shifter for each of the main demand sectors should be included.

Demand share by each sector from 1960 until 2003 is reported in Figure 22, adapted from Graedel et al. (Kavlak and Graedel 2013b). The data is for US only, but are assumed to be similar also at global level.

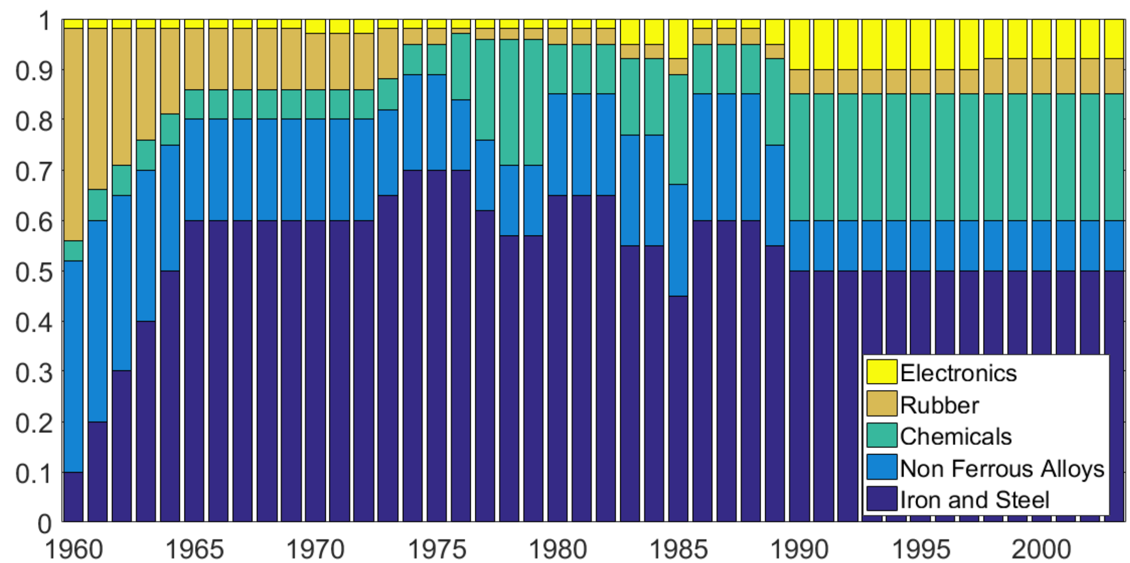


Figure 22: Tellurium demand by sector. Adapted from data by (Kavlak and Graedel 2013b)

In the time span considered, steel production and non-ferrous metal production are the two dominating sectors, covering around 60-70% of overall demand. In most recent year, however, tellurium demand has shifted towards the electronics sector, mainly driven by thermoelectric materials at first and thin film PV cells more recently. Four demand shifters are therefore collected, one related to steel production, one to non-ferrous metal production, one for cadmium telluride solar cell and a last one for thermoelectric devices. While the first two are related to supply quantities, the two referring to electronics devices represents efficiencies and therefore are linked to technological improvements. The four time series are presented in Figure 23. Steel production is obtained from USGS steel DS140 time series (US Geological Survey Iron and steel statistics 2016). Non-ferrous metal production is also compiled with USGS DS140 time series, coupling aluminum, copper, lead, nickel, tin and zinc (US Geological Survey aluminum statistics 2016)(US Geological Survey copper statistics 2016)(US Geological Survey Lead Statistics 2016)(US Geological Survey Nickel Statistics 2016)(US Geological Survey Tin Statistics 2016)(US Geological Survey Zinc Statistics

2016). CdTe cells efficiencies are taken from the “Best Research-Cell Efficiencies” graph by the National Renewable Energy Laboratories and represent the top recorded efficiency in the labs (National Renewable Energy Laboratory n.d.). Finally, thermoelectric efficiency are taken from (Heremans et al. 2013) and represented by ZT, a dimensionless figure of merit of thermoelectric materials. Greater ZT, higher the efficiency. For the details, the reader is send to the source article. It is decided not to include in the model both efficiency related data. Thermoelectric materials efficiency is discarded due to its poor quality: by showing only two different values in the time span considered, it is not considered reliable to include it as an indicator. PV cells’ efficiency, instead, is not included due to time consideration. Although cadmium telluride cells have been invented and implemented starting from the mid ‘70s, it is only in recent years that they have been mass produced, starting in 2002 when First Solar entered the market. CdTe PV cells’ efficiency, is therefore not considered a good demand shifter for the period analyzed.

World GDP and S&P 500 index are also included and their trends shown in Figure 22.E and 22.F respectively.

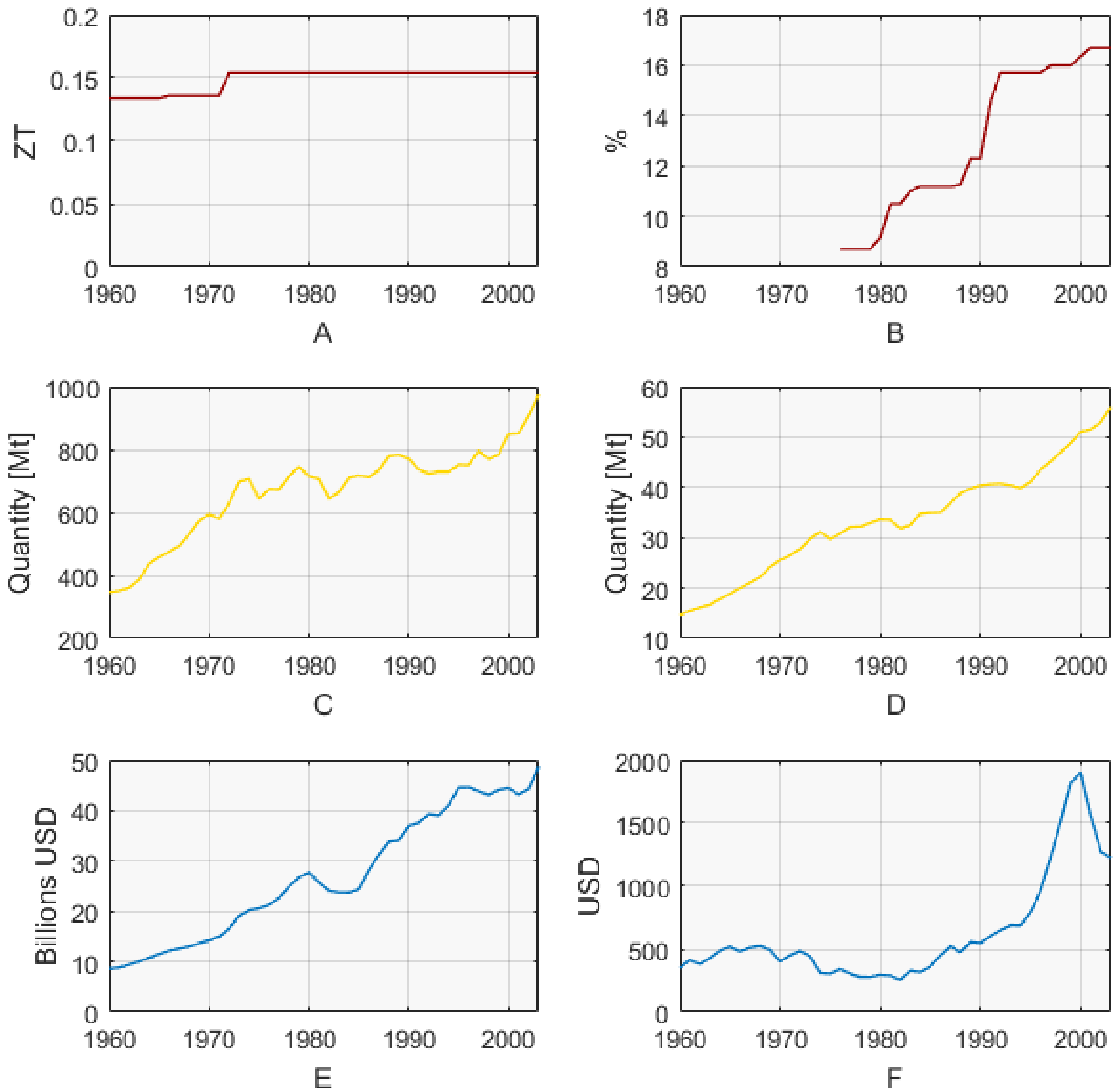


Figure 23: Tellurium demand shifters. A and B are related with thermoelectrical materials and CdTe PV cells efficiencies respectively. C represents supply of steel, while D supply of non-ferrous metals. E is world GDP, while F is S&P500 index.

Common Shifters

Time is the sole common shifter included. Again, on the supply side this can be seen as expressing extraction technological improvement, while on the demand it could represent an unobserved demand shifter such as population growth.

5.2.2 Results & Conclusion

Ordinary Least Squares Model

The number of variables is reduced, as in the case of indium. Interest rate 1y and G7 IP are selected and the complete equations are as follow.

$$Q_t^s = c + \alpha_1 P_t + \alpha_{2a}(Q_{Cu,conc})_t + \alpha_{2b}(G7\ IP)_t + \alpha_{2c}(Int.10y)_t + \alpha_3 Y_t + \varepsilon_t^s \qquad (5.2.2.1)$$

$$Q_t^d = c + \beta_1 P_t + \beta_{2a}(Q_{Steel})_t + \beta_{2b}(Q_{Non\ ferr})_t + \beta_{2c}(S\&P500)_t + \beta_3 Y_t + \varepsilon_t^s \qquad (5.2.2.2)$$

Regression using ordinary least squares is performed and the results reported in Table 14 and 15.

Table 14: OLS model summary for tellurium supply

	Dependent variable:
	Te_quantity
Te_price	−0.090
	(0.073)
Cu_concentrate	1.687***
	(0.607)
G7_IP	−0.006
	(0.011)
Interest1y	0.007
	(0.009)
Year	−0.029*
	(0.016)
Constant	37.450
	(31.245)
Observations	44
R^2	0.281
Adjusted R^2	0.186
Residual Std. Error	0.155 (df = 38)
F Statistic	2.965** (df = 5; 38)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 15: OLS model summary for tellurium demand

	Dependent variable:
	Te_quantity
Te_price	−0.260**
	(0.097)
Steel_production	−1.212
	(0.800)
Nonferrous_production	1.355
	(1.017)
SP500	−0.094
	(0.082)
World_GDP	0.777*
	(0.395)
Year	−0.045**
	(0.019)
Constant	74.061**
	(29.959)
Observations	44
R^2	0.265
Adjusted R^2	0.146
Residual Std. Error	0.159 (df = 37)
F Statistic	2.226* (df = 6; 37)
Note:	*p<0.1; **p<0.05; ***p<0.01

Again, white estimator is used to correct small sample proprieties. The estimation and inference of demand and supply elasticity are as follow:

- Supply: -0.09, 95% CI [-0.22,0.04]
- Demand: -0.26, 95% CI [-0.46, -0.06]

As in the case of indium, price coefficients are very small. It is therefore argued that both supply and demand are price inelastic.

Autoregressive Distributed Lag Model

Stepwise regression is developed, using the OLS model as starting point. In the supply equation, G7 IP index and interest rate are eliminated, while for what concerns demand, world GDP is the only variable included, together price, which is always forced into the models.

Residuals of the stepwise regression are analyzed and serial autocorrelation verified for both supply and demand equations, as seen in Figure 24 and Figure 25.

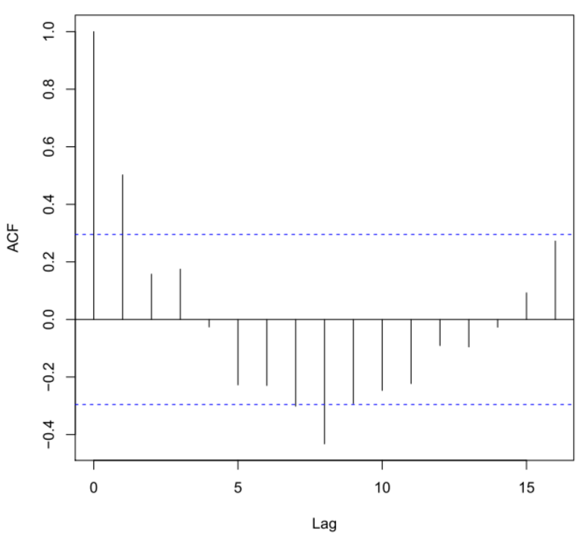


Figure 24: Autocorrelation of residuals of stepwise regression of tellurium supply

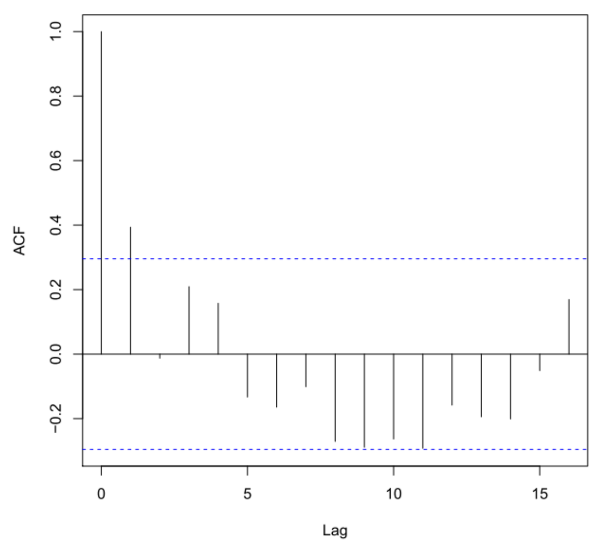


Figure 25: Autocorrelation of residuals of stepwise regression of tellurium demand

Once again, VAR model suggest to include only the first order of lag. Among all possible combinations, the model including first lag of both tellurium and copper concentrates supply shows the best adjusted R^2. Table 16 and 17 show the summary of the regression of the selected model.

Table 16: ARDL model summary for tellurium supply

	Dependent variable:
	Te_quantity
Year	−0.024**
	(0.011)
Te_price	−0.003
	(0.068)
Cu_concentrate	1.899***
	(0.603)
Teq1	0.521***
	(0.144)
Cuc1	−0.873
	(0.682)
Constant	32.893**
	(15.741)
Observations	43
R^2	0.471
Adjusted R^2	0.399
Residual Std. Error	0.134 (df = 37)
F Statistic	6.586*** (df = 5; 37)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 17: ARDL model summary for tellurium demand

	Dependent variable:
	Te_quantity
Year	−0.022*
	(0.011)
Te_price	−0.140
	(0.090)
World_GDP	0.528*
	(0.271)
Teq1	0.429***
	(0.142)
Constant	30.837**
	(14.843)
Observations	43
R^2	0.363
Adjusted R^2	0.295
Residual Std. Error	0.145 (df = 38)
F Statistic	5.403*** (df = 4; 38)
Note:	*p<0.1; **p<0.05; ***p<0.01

From Table 16 and 17 the improvements of the model are illustrated: the adjusted R^2 increasing from 0.18 to 0.40 for the supply model and from to 0.15 to 0.30 for the demand one.

In order to check if autocorrelation is still present the residual of the ARDL regression are analyzed and the results shown in Figure 26 and 27. While for supply, serial correlation seems to be vanished, for what concerns demand this seems not to be true. While the residuals of stepwise regression showed autocorrelation at the first lag, in this case autocorrelation seems to occur at the second lag. However, given the fact that the autocorrelation value is just above the 0.3 and considering that the number of variables should be contained, it is not decided to add greater order of lag in the model.

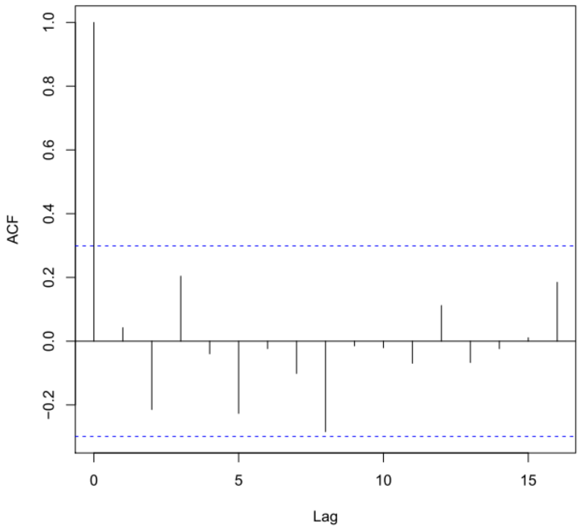


Figure 26: Autocorrelation of residuals of ARDL model for tellurium supply

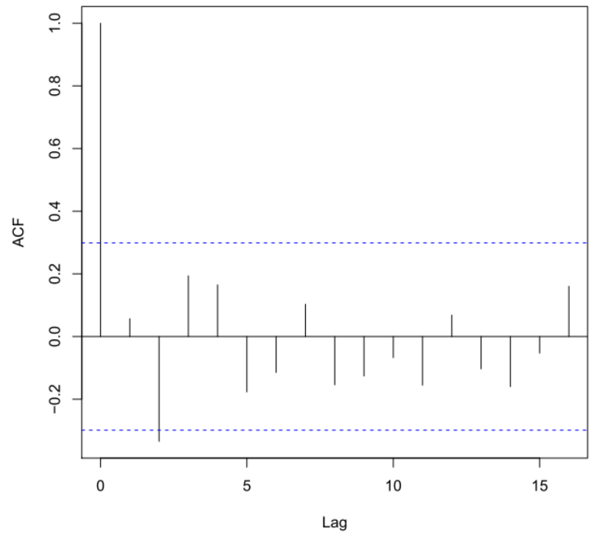


Figure 27: Autocorrelation of residuals of ARDL model for tellurium demand

White estimator is used to correct small sample proprieties and long run elasticity calculated according to Equation 3.2.2.3.

Estimation and inference of long run elasticity:

- Supply: -0.01; 95% CI [-0.25, 0.23]
- Demand: -0.24; 95% CI [-0.54, 0.05]

Similarly to the OLS model results, price coefficients are very small and the ranges cover zero. It is therefore argued that both supply and demand are price inelastic.

Instrumental Variable Model

ARDL model is used as starting point for the IV one. Supply shifters are used as instruments of the demand equation, and demand shifters for the supply one.

The results of the two regressions are shown here, with Table 18 and 19 referring to supply, and Table 20 and 21 to demand.

Table 18: First stage of IV model summary for tellurium supply

	Dependent variable:
	Te_price
Year	−0.034
	(0.027)
Cu_concentrate	−2.058
	(1.404)
Teq1	−0.621*
	(0.330)
CuC1	3.394**
	(1.536)
Constant	60.808
	(36.328)
Observations	43
R^2	0.138
Adjusted R^2	0.048
Residual Std. Error	0.321 (df = 38)
F Statistic	1.525 (df = 4; 38)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 19: Second stage of IV model summary for tellurium supply

	Dependent variable:
	Te_quantity
Te_price	0.034
	(0.102)
Year	−0.022*
	(0.012)
Cu_concentrate	1.976***
	(0.626)
Teq1	0.544***
	(0.152)
CuC1	−1.000
	(0.732)
Constant	30.610*
	(16.464)
Observations	43
R^2	0.467
Adjusted R^2	0.394
Residual Std. Error	0.135 (df = 37)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 20: First stage of IV model summary for tellurium demand

	Dependent variable:
	Te_price
Year	−0.079***
	(0.015)
World_GDP	1.928***
	(0.373)
Teq1	−0.557**
	(0.236)
Constant	111.642***
	(19.515)
Observations	43
R^2	0.423
Adjusted R^2	0.379
Residual Std. Error	0.259 (df = 39)
F Statistic	9.539*** (df = 3; 39)
Note:	*p<0.1; **p<0.05; ***p<0.01

Table 21: Second stage of IV model summary for tellurium demand

	Dependent variable:
	Te_quantity
Te_price	−0.646**
	(0.292)
Year	−0.062**
	(0.026)
World_GDP	1.504**
	(0.631)
Teq1	0.147
	(0.242)
Constant	87.364**
	(35.837)
Observations	43
R^2	−0.171
Adjusted R^2	−0.294
Residual Std. Error	0.197 (df = 38)
Note:	*p<0.1; **p<0.05; ***p<0.01

Estimation and inference of long run elasticity:

- Supply: -0.08; 95% CI [-0.49, 0.64]
- Demand: -0.76; 95% CI [-1.55, 0.03]

Once again, three tests are performed in order to validate the IV mode compared to the ARDL one. The summary of the tests is reported in Table 22.

Table 22: IV model tests summary for tellurium

	Supply	Implication	Demand	Implication
F Test	p<0.01	Instruments are correlated with price	p=0.48	Instruments show little correlation with price
Sargan–Hansen test for exogeneity	Not necessary		p=0.99	No statistical evidence of endogeneity of instruments
Wu-Hausman test	p=0.71	ARDL as con-sistent as IV	p<0.01	ARDL is statis-tically not as consistent as IV

As shown in Table 22, for what concerns the F test, two different result are obtained. Demand shifters correlate well with price and therefore they are good instruments for the supply equation. On the other hand, supply shifters do not correlate well with price and therefore demand instruments are weak. This is not a big issue since the goal of the analysis is to determine price elasticity of supply, not of demand. However, it should be noted how forcing bad instruments into the demand equation causes both R^2 and adjusted R^2 to be negative, which means that a horizontal line would fit the available data even better.

The Sargan-Hansen test is not needed for the supply equation since there is only one instrument (world GDP) for one endogenous variable (price) and therefore the problem of over identifying restrictions is not present. For what concerns demand, the test is performed and the p value greater than the significance level (0.05) suggests that the null hypothesis cannot be rejected. There is no statistical evidence that suggest endogeneity among the chosen instruments in the demand equation.

The Wu-Hausman test provides different results in the supply and demand cases. For what concerns supply, the large p value suggests that the null hypothesis cannot be rejected. This implies that there is no statistical evidence that IV model provides a better estimation compared to ARDL model. Moreover, given that goodness of instrument has been verified, endogeneity of price is considered responsible for this. For what concerns demand, the result of this test is irrelevant given that weakness of instruments have been verified with the F test.

Summary of three methods

In Table 23, a summary of the results of the three methods is presented. As for the case of indium, ARDL is found to be the most consistent among the three. Supply is argued to be price inelastic, given the small estimated value of price coefficient.

Table 23: Summary of supply elasticity results for tellurium

	α_1	Standard Error	95% Confidence Interval
OLS	-0.09	0.07	(-0.22, 0.04)
ARDL	-0.01	0.12	(-0.25, 0.23)
IV	0.08	0.29	(-0.49, 0.64)

Discussion

Potential and actual supply of indium are plotted in order to verify whether the carrier metal-zinc-is responsible for the inelastic behavior of supply. The trend is shown in Figure 28. As for the case of indium, the potential supply time series is obtained considering 100% recovery of all tellurium available in copper ores. For this purpose, concentration ratio of 105 ppm is used, obtained from a survey among copper refining facilities (Green 2006).

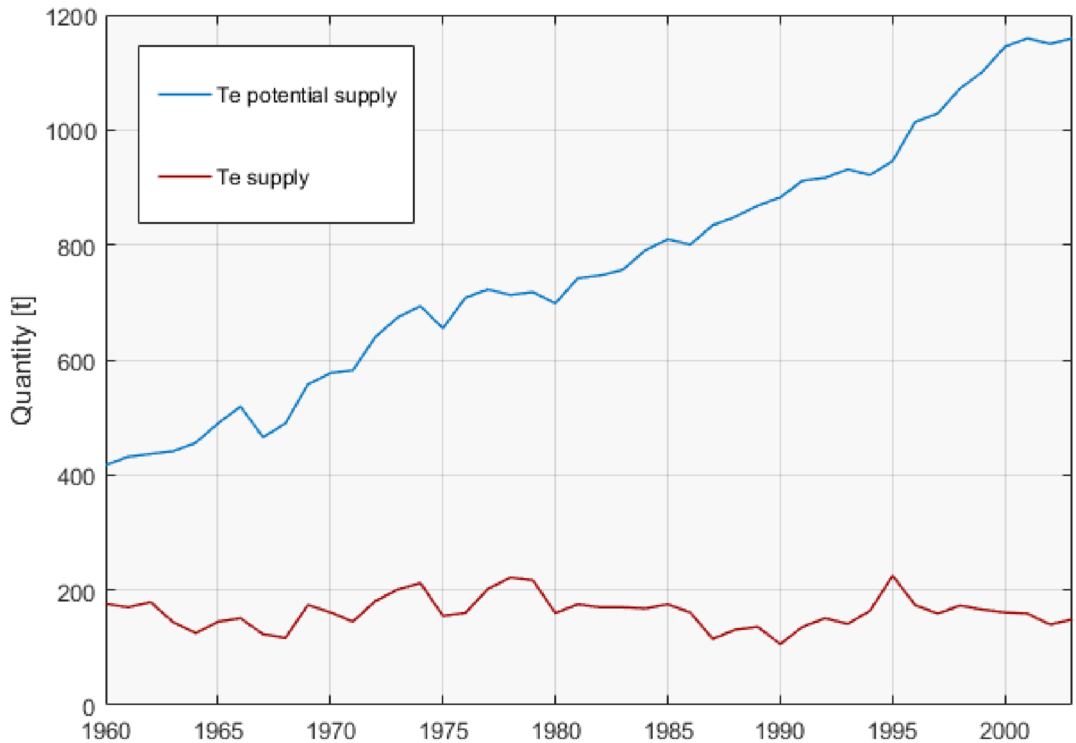


Figure 28: Tellurium supply and maximum potential supply from Cu concentrates (1960-2003)

It is argued that copper concentrate supply is not limiting the production of tellurium. As seen in Figure 28, tellurium supply potential has always been at least twice the actual production and increased constantly from 1960 until 2003.

As supplementary evidences of copper concentrate not limiting supply of tellurium, cointegration of Cu concentrate supply and Te supply is analyzed. Cointegration is statistical propriety of time series, somehow similar to correlation which however account for the time dimension. It can be tested using the Engle-Granger test, where the null hypothesis states that the different time series are not cointegrated. The p value is found to be larger than the significant interval (0.05) and therefore the null hypothesis cannot be rejected. There is no significant statistical evidence of cointergration between tellurium supply and copper concentrate supply.

Moreover, as stated in “Global anthropogenic tellurium cycles for 1940-2010 “ article by Greaedel et al., from 2003 to 2010 Te supply have increase 3-fold while copper concentrate supply increased by only 20%. In addition to this, in the same time period price of tellurium increased 5-fold, evidencing a higher elasticity.

Other factors must therefore be responsible for the observed inelasticity. Lack of a sport market able to respond quickly to changes in supply and demand and a potentially oligopolistic competition are believed to be major forces in play. In particular, in the case of oligopolistic competition, price is not determined by interactions between supply and demand but rather set by a very limited number of suppliers and this may explain alone the inelastic supply observed.
Further studies are needed in order to be able to identify and quantify all the different factors that may have driven supply inelasticity of tellurium between 1960 and 2003.

6 Conclusions and Future Work

The first aim of this work is to classify different carrier-byproduct pairs according to their criticality. The classification is done in a 2D matrix, with byproduct fraction and value ratio as the two dimensions. Byproduct fraction, which represents the amount of primary production obtained as byproduct, is linked with consumers' concerns. Value ratio instead represents how valuable the byproduct is for the mining company and therefore reflects producers' concerns. Both the qualitative and quantitative analyses performed identify five main groups of which two are considered critical for consumers, one for suppliers, one for both and one for none. In particular, PV related metals-such as cadmium, gallium, indium, selenium and tellurium all fall within the "consumers' high criticality" region of the matrix.

In the second part of the study, two PV metals are analyzed in order to assess whether supply is elastic. Tellurium and indium are chosen due to their large use in thin-film photovoltaics cells and availability of data. Price elasticity of supply is evaluated through econometric analysis, which includes ordinary least squares, autoregressive distributed lag and instrument variable models. The results suggest an inelastic supply for both byproduct metals. In the case of indium, limitation set by the supply of the carrier zinc is found to be the cause of such inelasticity. This is in line with the arguments in current literature that supply limitation of the byproduct imposed by the carrier may result in an inelastic supply of the companion metal. For tellurium, however, this seems not to be the case. The recovered amount from copper concentrates is currently smaller than 15%of the maximum supply potential from this carrier. Non-transparent trading, monopolistic character of supply and other factors are expected to be the major causes of supply inelasticity.

The outcome from categorization and the two case studies will be useful for decision making of both metals producers and consumers. For example, although it is believed that tellurium is one of the more critical metals, its supply risk should not be in the immediate concerns for CdTe PV cells manufacturers. The actual supply, in fact, is not reaching maximum supply potential from

copper ores and other sources such as lead and primary production exist. In addition, the fact that tellurium is supply inelastic suggest that a better exchange market be established.
Moreover, the problem that most of the byproduct metals are also minor metals and therefore are produced in a limited number of countries, should be tackled. The Fanya scam is an example of concentrated supply in China and lacking of government supervision (Haeyood 2015). In order to develop a healthy market mechanism of minor metals, international cooperation should be implemented so that metals can be traded in more transparent ways. This will partially mitigate material criticality concerns of these metals and encourage continuing development and greater industrial scale application of renewable technologies.

For what concerns future work, in the short term similar econometric models will be used to study other byproduct metals belonging to the "High criticality" group, such as selenium.
Following this, other models will be developed in order to analyze the different groups identified from the categorization process. For the medium criticality group, germanium will likely be chosen due to its being obtained from two peculiar carriers, namely zinc and coal. The wo carrier materials have very different applications and market behaviors and it is interesting to see how the interaction between these two carriers affects the supply of germanium. For what concerns the coproduct group, a method will be developed from the mining companies point of view. This method will look at company specific characteristics such as production and manufacturing costs, breakdown of revenues, etc.
In the long term, the impact of current human activities on the byproduct availability will be evaluated in details. Some specific strategies will be assessed from the materials system perspective such as increased recycling, substitution and improvement in extraction technologies.
These will inform decision making in raw material extraction, sustainable metal use and waste management.

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8 Appendix

Byproduct fraction, quantity ratio and price for all carrier-byproduct pairs analyzed.

Carrier	Byproduct	Byproduct fraction	Quantity Ratio	Carrier price (2015 $/lb)	Byproduct price (2015 $/lb)
Pb	Sb	40.0%	0.0018	0.83	3.44
Cu	Sb	0.0%	0.00014	2.7	3.44
Au	Sb	4.0%	477	17062	3.44
Sn	Sb	8.0%	1	7	3.44
Cu	Bi	5.5%	0.00002	2.7	7.5
Pb	Bi	33.0%	0.0068	0.83	7.5
Mo	Bi	3.0%	0.22	8.1	7.5
Sn	Bi	5.5%	0.0475	7	7.5
W	Bi	43.0%	0.417	18.3	7.5
Zn	Bi	3.5%	0.000097	0.87	7.5
Zn	Cd	100.0%	0.0055	0.87	0.48
As	Co	2.0%	0.197	0.38	13.1
Cu	Co	35.0%	0.1	2.7	13.1
Ni	Co	50.0%	0.043	5.73	13.1
Au	Cu	2.0%	7980	17062	2.7
Pb	Cu	2.0%	0.092	0.83	2.7
Ni	Cu	5.0%	0.50	5.73	2.7
PGM	Cu	0.1%	62.6	13348	2.7
Ag	Cu	0.1%	20.06	233	2.7
Al	Ga	98.0%	0.00018	0.88	133.9
Zn	Ga	2.0%	0.000003	0.87	133.9

Coal	Ge	40.0%	0.000002	0.022	798.6
Zn	Ge	60.0%	0.00007	0.87	798.6
Cu	Au	12.0%	0.00004	2.7	17062
Ag	Au	0.5%	0.00079	233	17062
Ni	Au	0.1%	0.000022	5.73	17062
PGM	Au	0.3%	0.031	13348	17062
Zn	Au	1.0%	0.000028	0.87	17062
Zn	In	95.0%	0.00004	0.87	227
Sn	In	3.8%	0.0112	7	227
Cu	Mo	61.0%	0.0185	2.7	8.1
Ta	Nb	1.0%	0.182	107	29.3
Sn	Nb	2.0%	0.007	7	29.3
Cu/Mo	Re	71.0%	0.068	8.1	1316
Cu	Re	29.0%	0.00011	2.7	1316
Cu	Se	100.0%	0.0002	2.7	22.8
Pb	Se	0.0%	0.000011	0.83	22.8
Cu	Ag	20.0%	0.00017	2.7	233
Pb/Zn	Ag	35.0%	0.002	0.85	233
Au	Ag	13.0%	1.25	17062	233
Nb	Ta	27.0%	0.036	29.3	107
Sn	Ta	33.0%	0.029	7	107
Pb	Te	8.0%	0.00059	0.83	40.4
Cu	Te	76.5%	0.000058	2.7	40.4
Cu	Zn	4.0%	0.026	2.7	0.87
Ag	Zn	3.0%	182	233	0.87
Au	Zn	2.0%	7460	17062	0.87